Rational Numbers

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1.1 Introduction

In Mathematics, we frequently come across simple equations to be solved. For example, the equation

$$x + 2 = 13 \qquad (1)$$

is solved when $x = 11$, because this value of x satisfies the given equation. The solution 11 is a **natural number**. On the other hand, for the equation

$$x + 5 = 5 \qquad (2)$$

the solution gives the **whole number** 0 (zero). If we consider only natural numbers, equation (2) cannot be solved. To solve equations like (2), we added the number zero to the collection of natural numbers and obtained the whole numbers. Even whole numbers will not be sufficient to solve equations of type

$$x + 18 = 5 \qquad (3)$$

Do you see 'why'? We require the number –13 which is not a whole number. This led us to think of **integers, (positive and negative)**. Note that the positive integers correspond to natural numbers. One may think that we have enough numbers to solve all simple equations with the available list of integers. Now consider the equations

$$2x = 3 \qquad (4)$$
$$5x + 7 = 0 \qquad (5)$$

for which we cannot find a solution from the integers. (Check this)

We need the numbers $\dfrac{3}{2}$ to solve equation (4) and $\dfrac{-7}{5}$ to solve equation (5). This leads us to the collection of **rational numbers**.

We have already seen basic operations on rational numbers. We now try to explore some properties of operations on the different types of numbers seen so far.

1.2 Properties of Rational Numbers

1.2.1 Closure

(i) Whole numbers

Let us revisit the closure property for all the operations on whole numbers in brief.

Operation	Numbers	Remarks
Addition	$0 + 5 = 5$, a whole number $4 + 7 = \ldots$. Is it a whole number? In general, $a + b$ is a whole number for any two whole numbers a and b.	Whole numbers are closed under addition.
Subtraction	$5 - 7 = -2$, which is not a whole number.	Whole numbers are **not** closed under subtraction.
Multiplication	$0 \times 3 = 0$, a whole number $3 \times 7 = \ldots$. Is it a whole number ? In general, if a and b are any two whole numbers, their product ab is a whole number.	Whole numbers are closed under multiplication.
Division	$5 \div 8 = \dfrac{5}{8}$, which is not a whole number.	Whole numbers are **not** closed under division.

Check for closure property under all the four operations for natural numbers.

(ii) Integers

Let us now recall the operations under which integers are closed.

Operation	Numbers	Remarks
Addition	$-6 + 5 = -1$, an integer Is $-7 + (-5)$ an integer? Is $8 + 5$ an integer? In general, $a + b$ is an integer for any two integers a and b.	Integers are closed under addition.
Subtraction	$7 - 5 = 2$, an integer Is $5 - 7$ an integer? $-6 - 8 = -14$, an integer	Integers are closed under subtraction.

	$-6-(-8)=2$, an integer Is $8-(-6)$ an integer? In general, for any two integers a and b, $a-b$ is again an integer. Check if $b-a$ is also an integer.	
Multiplication	$5 \times 8 = 40$, an integer Is -5×8 an integer? $-5 \times (-8) = 40$, an integer In general, for any two integers a and b, $a \times b$ is also an integer.	Integers are closed under multiplication.
Division	$5 \div 8 = \dfrac{5}{8}$, which is not an integer.	Integers are **not** closed under division.

You have seen that whole numbers are closed under addition and multiplication but not under subtraction and division. However, integers are closed under addition, subtraction and multiplication but not under division.

(iii) Rational numbers

Recall that a number which can be written in the form $\dfrac{p}{q}$, where p and q are integers and $q \neq 0$ is called a **rational number**. For example, $-\dfrac{2}{3}, \dfrac{6}{7}, \dfrac{9}{-5}$ are all rational numbers. Since the numbers $0, -2, 4$ can be written in the form $\dfrac{p}{q}$, they are also rational numbers. (Check it!)

(a) You know how to add two rational numbers. Let us add a few pairs.

$$\frac{3}{8} + \frac{(-5)}{7} = \frac{21 + (-40)}{56} = \frac{-19}{56} \qquad \text{(a rational number)}$$

$$\frac{-3}{8} + \frac{(-4)}{5} = \frac{-15 + (-32)}{40} = \dots \qquad \text{Is it a rational number?}$$

$$\frac{4}{7} + \frac{6}{11} = \dots \qquad \text{Is it a rational number?}$$

We find that sum of two rational numbers is again a rational number. Check it for a few more pairs of rational numbers.

We say that *rational numbers are closed under addition. That is, for any two rational numbers a and b, a + b is also a rational number* .

(b) Will the difference of two rational numbers be again a rational number?
We have,

$$\frac{-5}{7} - \frac{2}{3} = \frac{-5 \times 3 - 2 \times 7}{21} = \frac{-29}{21} \qquad \text{(a rational number)}$$

$$\frac{5}{8} - \frac{4}{5} = \frac{25 - 32}{40} = ...$$ Is it a rational number?

$$\frac{3}{7} - \left(\frac{-8}{5}\right) = ...$$ Is it a rational number?

Try this for some more pairs of rational numbers. We find that *rational numbers are closed under subtraction. That is, for any two rational numbers a and b, a – b is also a rational number.*

(c) Let us now see the product of two rational numbers.

$$\frac{-2}{3} \times \frac{4}{5} = \frac{-8}{15}; \frac{3}{7} \times \frac{2}{5} = \frac{6}{35}$$ (both the products are rational numbers)

$$-\frac{4}{5} \times \frac{-6}{11} = ...$$ Is it a rational number?

Take some more pairs of rational numbers and check that their product is again a rational number.

We say that *rational numbers are closed under multiplication. That is, for any two rational numbers a and b, a × b is also a rational number.*

(d) We note that $\dfrac{-5}{3} \div \dfrac{2}{5} = \dfrac{-25}{6}$ (a rational number)

$\dfrac{2}{7} \div \dfrac{5}{3} = .$. Is it a rational number? $\dfrac{-3}{8} \div \dfrac{-2}{9} = ..$. Is it a rational number?

Can you say that rational numbers are closed under division?

We find that for any rational number a, $a \div 0$ is **not defined**.

So rational numbers are **not closed** under division.

However, if we exclude zero then the collection of, all other rational numbers is closed under division.

TRY THESE

Fill in the blanks in the following table.

Numbers	Closed			
	addition	**subtraction**	**multiplication**	**division**
Rational numbers	Yes	Yes	...	No
I	...	Yes	...	No
Whole numbers	...	...	Yes	...
Natural numbers	...	No	...	...

1.2.2 Commutativity

(i) Whole numbers

Recall the commutativity of different operations for whole numbers by filling the following table.

Operation	Numbers	Remarks
Addition	$0 + 7 = 7 + 0 = 7$ $2 + 3 = ... + ... = ...$ For any two whole numbers a and b, $a + b = b + a$	Addition is commutative.
Subtraction	....	Subtraction is not commutative.
Multiplication	....	Multiplication is commutative.
Division	....	Division is not commutative.

Check whether the commutativity of the operations hold for natural numbers also.

(ii) Integers

Fill in the following table and check the commutativity of different operations for integers:

Operation	Numbers	Remarks
Addition	....	Addition is commutative.
Subtraction	Is $5 - (-3) = -3 - 5$?	Subtraction is not commutative.
Multiplication	....	Multiplication is commutative.
Division	....	Division is not commutative.

(iii) Rational numbers

(a) Addition

You know how to add two rational numbers. Let us add a few pairs here.

$$\frac{-2}{3} + \frac{5}{7} = \frac{1}{21} \text{ and } \frac{5}{7} + \left(\frac{-2}{3}\right) = \frac{1}{21}$$

So, $$\frac{-2}{3} + \frac{5}{7} = \frac{5}{7} + \left(\frac{-2}{3}\right)$$

Also, $$\frac{-6}{5} + \left(\frac{-8}{3}\right) = ... \text{ and } \frac{-8}{3} + \left(\frac{-6}{5}\right) = ...$$

Is $$\frac{-6}{5} + \left(\frac{-8}{3}\right) = \left(\frac{-8}{3}\right) + \left(\frac{-6}{5}\right)?$$

Is $\qquad \dfrac{-3}{8} + \dfrac{1}{7} = \dfrac{1}{7} + \left(\dfrac{-3}{8}\right)$?

You find that two *rational numbers can be added in any order. We say that addition is commutative for rational numbers. That is, for any two rational numbers a and b, a + b = b + a.*

(b) Subtraction

Is $\qquad \dfrac{2}{3} - \dfrac{5}{4} = \dfrac{5}{4} - \dfrac{2}{3}$?

Is $\qquad \dfrac{1}{2} - \dfrac{3}{5} = \dfrac{3}{5} - \dfrac{1}{2}$?

You will find that subtraction is not commutative for rational numbers.

Note that subtraction is not commutative for integers and integers are also rational numbers. So, subtraction will not be commutative for rational numbers too.

(c) Multiplication

We have, $\qquad \dfrac{-7}{3} \times \dfrac{6}{5} = \dfrac{-42}{15} = \dfrac{6}{5} \times \left(\dfrac{-7}{3}\right)$

Is $\qquad \dfrac{-8}{9} \times \left(\dfrac{-4}{7}\right) = \dfrac{-4}{7} \times \left(\dfrac{-8}{9}\right)$?

Check for some more such products.

You will find that *multiplication is commutative for rational numbers*.

In general, a × b = b × a for any two rational numbers a and b.

(d) Division

Is $\qquad \dfrac{-5}{4} \div \dfrac{3}{7} = \dfrac{3}{7} \div \left(\dfrac{-5}{4}\right)$?

You will find that expressions on both sides are not equal.

So division is **not commutative** for rational numbers.

TRY THESE

Complete the following table:

Numbers	Commutative for			
	addition	subtraction	multiplication	division
Rational numbers	Yes	...	...	...
Integers	...	No	...	...
Whole numbers	...	...	Yes	...
Natural numbers	...	...	...	No

1.2.3 Associativity

(i) Whole numbers

Recall the associativity of the four operations for whole numbers through this table:

Operation	Numbers	Remarks
Addition	....	Addition is associative
Subtraction	....	Subtraction is **not** associative
Multiplication	Is $7 \times (2 \times 5) = (7 \times 2) \times 5$? Is $4 \times (6 \times 0) = (4 \times 6) \times 0$? For any three whole numbers a, b and c $a \times (b \times c) = (a \times b) \times c$	Multiplication is associative
Division	....	Division is **not** associative

Fill in this table and verify the remarks given in the last column.
Check for yourself the associativity of different operations for natural numbers.

(ii) Integers

Associativity of the four operations for integers can be seen from this table

Operation	Numbers	Remarks
Addition	Is $(-2) + [3 + (-4)]$ $= [(-2) + 3)] + (-4)$? Is $(-6) + [(-4) + (-5)]$ $= [(-6) + (-4)] + (-5)$? For any three integers a, b and c $a + (b + c) = (a + b) + c$	Addition is associative
Subtraction	Is $5 - (7 - 3) = (5 - 7) - 3$?	Subtraction is **not** associative
Multiplication	Is $5 \times [(-7) \times (-8)$ $= [5 \times (-7)] \times (-8)$? Is $(-4) \times [(-8) \times (-5)]$ $= [(-4) \times (-8)] \times (-5)$? For any three integers a, b and c $a \times (b \times c) = (a \times b) \times c$	Multiplication is associative
Division	Is $[(-10) \div 2] \div (-5)$ $= (-10) \div [2 \div (-5)]$?	Division is **not** associative

(iii) Rational numbers

(a) Addition

We have $\dfrac{-2}{3}+\left[\dfrac{3}{5}+\left(\dfrac{-5}{6}\right)\right]=\dfrac{-2}{3}+\dfrac{-7}{30}=\dfrac{-27}{30}=\dfrac{-9}{10}$

$\left[\dfrac{-2}{3}+\dfrac{3}{5}\right]+\left(\dfrac{-5}{6}\right)=\dfrac{-1}{15}+\dfrac{-5}{6}=\dfrac{-27}{30}=\dfrac{-9}{10}$

So, $\dfrac{-2}{3}+\left[\dfrac{3}{5}+\left(\dfrac{-5}{6}\right)\right]=\left[\dfrac{-2}{3}+\dfrac{3}{5}\right]+\left(\dfrac{-5}{6}\right)$

Find $\dfrac{-1}{2}+\left[\dfrac{3}{7}+\left(\dfrac{-4}{3}\right)\right]$ and $\left[\dfrac{-1}{2}+\dfrac{3}{7}\right]+\left(\dfrac{-4}{3}\right)$. Are the two sums equal?

Take some more rational numbers, add them as above and see if the two sums are equal. We find that *addition is associative for rational numbers. That is, for any three rational numbers a, b and c, a + (b + c) = (a + b) + c.*

(b) Subtraction

You already know that subtraction is not associative for integers, then what about rational numbers.

Is $\dfrac{-2}{3}-\left[\dfrac{-4}{5}-\dfrac{1}{2}\right]=\left[\dfrac{2}{3}-\left(\dfrac{-4}{5}\right)\right]-\dfrac{1}{2}$?

Check for yourself.

Subtraction is **not associative** for rational numbers.

(c) Multiplication

Let us check the associativity for multiplication.

$$\dfrac{-7}{3}\times\left(\dfrac{5}{4}\times\dfrac{2}{9}\right)=\dfrac{-7}{3}\times\dfrac{10}{36}=\dfrac{-70}{108}=\dfrac{-35}{54}$$

$$\left(\dfrac{-7}{3}\times\dfrac{5}{4}\right)\times\dfrac{2}{9}=\ldots$$

We find that $\dfrac{-7}{3}\times\left(\dfrac{5}{4}\times\dfrac{2}{9}\right)=\left(\dfrac{-7}{3}\times\dfrac{5}{4}\right)\times\dfrac{2}{9}$

Is $\dfrac{2}{3}\times\left(\dfrac{-6}{7}\times\dfrac{4}{5}\right)=\left(\dfrac{2}{3}\times\dfrac{-6}{7}\right)\times\dfrac{4}{5}$?

Take some more rational numbers and check for yourself.

We observe that *multiplication is associative for rational numbers. That is for any three rational numbers a, b and c, a × (b × c) = (a × b) × c.*

(d) Division

Recall that division is not associative for integers, then what about rational numbers?

Let us see if $\dfrac{1}{2} \div \left[\dfrac{-1}{3} \div \dfrac{2}{5} \right] = \left[\dfrac{1}{2} \div \left(\dfrac{-1}{3} \right) \right] \div \dfrac{2}{5}$

We have, LHS $= \dfrac{1}{2} \div \left(\dfrac{-1}{3} \div \dfrac{2}{5} \right) = \dfrac{1}{2} \div \left(\dfrac{-1}{3} \times \dfrac{5}{2} \right)$ (reciprocal of $\dfrac{2}{5}$ is $\dfrac{5}{2}$)

$$= \dfrac{1}{2} \div \left(-\dfrac{5}{6} \right) = ...$$

RHS $= \left[\dfrac{1}{2} \div \left(\dfrac{-1}{3} \right) \right] \div \dfrac{2}{5}$

$$= \left(\dfrac{1}{2} \times \dfrac{-3}{1} \right) \div \dfrac{2}{5} = \dfrac{-3}{2} \div \dfrac{2}{5} = ...$$

Is LHS = RHS? Check for yourself. You will find that division is **not associative** for rational numbers.

TRY THESE

Complete the following table:

Numbers	Associative for			
	addition	subtraction	multiplication	division
Rational numbers	...	...	...	No
Integers	...	...	Yes	...
Whole numbers	Yes	...	...	...
Natural numbers	...	No	...	...

Example 1: Find $\dfrac{3}{7} + \left(\dfrac{-6}{11} \right) + \left(\dfrac{-8}{21} \right) + \left(\dfrac{5}{22} \right)$

Solution: $\dfrac{3}{7} + \left(\dfrac{-6}{11} \right) + \left(\dfrac{-8}{21} \right) + \left(\dfrac{5}{22} \right)$

$$= \dfrac{198}{462} + \left(\dfrac{-252}{462} \right) + \left(\dfrac{-176}{462} \right) + \left(\dfrac{105}{462} \right) \text{ (Note that 462 is the LCM of } 7, 11, 21 \text{ and } 22)$$

$$= \dfrac{198 - 252 - 176 + 105}{462} = \dfrac{-125}{462}$$

We can also solve it as.

$$\frac{3}{7}+\left(\frac{-6}{11}\right)+\left(\frac{-8}{21}\right)+\frac{5}{22}$$

$$=\left[\frac{3}{7}+\left(\frac{-8}{21}\right)\right]+\left[\frac{-6}{11}+\frac{5}{22}\right] \qquad \text{(by using commutativity and associativity)}$$

$$=\left[\frac{9+(-8)}{21}\right]+\left[\frac{-12+5}{22}\right] \quad \text{(LCM of 7 and 21 is 21 ; LCM of 11 and 22 is 22)}$$

$$=\frac{1}{21}+\left(\frac{-7}{22}\right) = \frac{22-147}{462} = \frac{-125}{462}$$

Do you think the properties of commutativity and associativity made the calculations easier?

Example 2: Find $\dfrac{-4}{5}\times\dfrac{3}{7}\times\dfrac{15}{16}\times\left(\dfrac{-14}{9}\right)$

Solution: We have

$$\frac{-4}{5}\times\frac{3}{7}\times\frac{15}{16}\times\left(\frac{-14}{9}\right)$$

$$=\left(-\frac{4\times3}{5\times7}\right)\times\left(\frac{15\times(-14)}{16\times9}\right)$$

$$=\frac{-12}{35}\times\left(\frac{-35}{24}\right)=\frac{-12\times(-35)}{35\times24}=\frac{1}{2}$$

We can also do it as.

$$\frac{-4}{5}\times\frac{3}{7}\times\frac{15}{16}\times\left(\frac{-14}{9}\right)$$

$$=\left(\frac{-4}{5}\times\frac{15}{16}\right)\times\left[\frac{3}{7}\times\left(\frac{-14}{9}\right)\right] \quad \text{(Using commutativity and associativity)}$$

$$=\frac{-3}{4}\times\left(\frac{-2}{3}\right) = \frac{1}{2}$$

1.2.4 The role of zero (0)

Look at the following.

$$2 + 0 = 0 + 2 = 2 \qquad \text{(Addition of 0 to a whole number)}$$

$$-5 + 0 = ... + ... = -5 \qquad \text{(Addition of 0 to an integer)}$$

$$\frac{-2}{7} + ... = 0 + \left(\frac{-2}{7}\right) = \frac{-2}{7} \qquad \text{(Addition of 0 to a rational number)}$$

You have done such additions earlier also. Do a few more such additions.

What do you observe? You will find that when you add 0 to a whole number, the sum is again that whole number. This happens for integers and rational numbers also.

In general,
$$a + 0 = 0 + a = a, \quad \text{where } a \text{ is a whole number}$$
$$b + 0 = 0 + b = b, \quad \text{where } b \text{ is an integer}$$
$$c + 0 = 0 + c = c, \quad \text{where } c \text{ is a rational number}$$

Zero is called the identity for the addition of rational numbers. It is the additive identity for integers and whole numbers as well.

1.2.5 The role of 1

We have,
$$5 \times 1 = 5 = 1 \times 5 \quad \text{(Multiplication of 1 with a whole number)}$$

$$\frac{-2}{7} \times 1 = ... \times ... = \frac{-2}{7}$$

$$\frac{3}{8} \times ... = 1 \times \frac{3}{8} = \frac{3}{8}$$

What do you find?

You will find that when you multiply any rational number with 1, you get back the same rational number as the product. Check this for a few more rational numbers. You will find that, $a \times 1 = 1 \times a = a$ for any rational number a.

We say *that 1 is the multiplicative identity for rational numbers.*

Is 1 the multiplicative identity for integers? For whole numbers?

THINK, DISCUSS AND WRITE

If a property holds for rational numbers, will it also hold for integers? For whole numbers? Which will? Which will not?

1.2.6 Negative of a number

While studying integers you have come across negatives of integers. What is the negative of 1? It is – 1 because $1 + (-1) = (-1) + 1 = 0$

So, what will be the negative of (–1)? It will be 1.

Also, $2 + (-2) = (-2) + 2 = 0$, so we say 2 is the **negative or additive inverse** of –2 and vice-versa. In general, for an integer a, we have, $a + (-a) = (-a) + a = 0$; so, a is the negative of $-a$ and $-a$ is the negative of a.

For the rational number $\frac{2}{3}$, we have,

$$\frac{2}{3} + \left(-\frac{2}{3}\right) = \frac{2 + (-2)}{3} = 0$$

Also,
$$\left(-\frac{2}{3}\right)+\frac{2}{3} = 0 \qquad \text{(How?)}$$

Similarly,
$$\frac{-8}{9}+\ldots = \ldots+\left(\frac{-8}{9}\right)= 0$$

$$\ldots+\left(\frac{-11}{7}\right) = \left(\frac{-11}{7}\right)+\ldots = 0$$

In general, for a rational number $\frac{a}{b}$, we have, $\frac{a}{b}+\left(-\frac{a}{b}\right)=\left(-\frac{a}{b}\right)+\frac{a}{b}= 0$. We say

that $-\frac{a}{b}$ *is the additive inverse of* $\frac{a}{b}$ *and* $\frac{a}{b}$ *is the additive inverse of* $\left(-\frac{a}{b}\right)$.

1.2.7 Reciprocal

By which rational number would you multiply $\frac{8}{21}$, to get the product 1? Obviously by

$\frac{21}{8}$, since $\frac{8}{21}\times\frac{21}{8}=1$.

Similarly, $\frac{-5}{7}$ must be multiplied by $\frac{7}{-5}$ so as to get the product 1.

We say that $\frac{21}{8}$ is the reciprocal of $\frac{8}{21}$ and $\frac{7}{-5}$ is the reciprocal of $\frac{-5}{7}$.

Can you say what is the reciprocal of 0 (zero)?

Is there a rational number which when multiplied by 0 gives 1? Thus, zero has no reciprocal.

We say that a rational number $\frac{c}{d}$ is called the **reciprocal** or **multiplicative inverse** of

another non-zero rational number $\frac{a}{b}$ if $\frac{a}{b}\times\frac{c}{d}=1$.

1.2.8 Distributivity of multiplication over addition for rational numbers

To understand this, consider the rational numbers $\frac{-3}{4}, \frac{2}{3}$ and $\frac{-5}{6}$.

$$\frac{-3}{4}\times\left\{\frac{2}{3}+\left(\frac{-5}{6}\right)\right\} = \frac{-3}{4}\times\left\{\frac{(4)+(-5)}{6}\right\}$$

$$= \frac{-3}{4}\times\left(\frac{-1}{6}\right) = \frac{3}{24}=\frac{1}{8}$$

Also
$$\frac{-3}{4}\times\frac{2}{3} = \frac{-3\times 2}{4\times 3}=\frac{-6}{12}=\frac{-1}{2}$$

And $$\frac{-3}{4} \times \frac{-5}{6} = \frac{5}{8}$$

Therefore $$\left(\frac{-3}{4} \times \frac{2}{3}\right) + \left(\frac{-3}{4} \times \frac{-5}{6}\right) = \frac{-1}{2} + \frac{5}{8} = \frac{1}{8}$$

Thus, $$\frac{-3}{4} \times \left\{\frac{2}{3} + \frac{-5}{6}\right\} = \left(\frac{-3}{4} \times \frac{2}{3}\right) + \left(\frac{-3}{4} \times \frac{-5}{6}\right)$$

> **Distributivity of Multi-plication over Addition and Subtraction.**
> For all rational numbers a, b and c,
> $a\,(b + c) = ab + ac$
> $a\,(b - c) = ab - ac$

TRY THESE

Find using distributivity. (i) $\left\{\frac{7}{5} \times \left(\frac{-3}{12}\right)\right\} + \left\{\frac{7}{5} \times \frac{5}{12}\right\}$ (ii) $\left\{\frac{9}{16} \times \frac{4}{12}\right\} + \left\{\frac{9}{16} \times \frac{-3}{9}\right\}$

Example 3: Write the additive inverse of the following:

(i) $\frac{-7}{19}$ (ii) $\frac{21}{112}$

> When you use distributivity, you split a product as a sum or difference of two products.

Solution:

(i) $\frac{7}{19}$ is the additive inverse of $\frac{-7}{19}$ because $\frac{-7}{19} + \frac{7}{19} = \frac{-7+7}{19} = \frac{0}{19} = 0$

(ii) The additive inverse of $\frac{21}{112}$ is $\frac{-21}{112}$ (Check!)

Example 4: Verify that $-(-x)$ is the same as x for

(i) $x = \frac{13}{17}$ (ii) $x = \frac{-21}{31}$

Solution: (i) We have, $x = \frac{13}{17}$

The additive inverse of $x = \frac{13}{17}$ is $-x = \frac{-13}{17}$ since $\frac{13}{17} + \left(\frac{-13}{17}\right) = 0$.

The same equality $\frac{13}{17} + \left(\frac{-13}{17}\right) = 0$, shows that the additive inverse of $\frac{-13}{17}$ is $\frac{13}{17}$

or $-\left(\frac{-13}{17}\right) = \frac{13}{17}$, i.e., $-(-x) = x$.

(ii) Additive inverse of $x = \frac{-21}{31}$ is $-x = \frac{21}{31}$ since $\frac{-21}{31} + \frac{21}{31} = 0$.

The same equality $\frac{-21}{31} + \frac{21}{31} = 0$, shows that the additive inverse of $\frac{21}{31}$ is $\frac{-21}{31}$,
i.e., $-(-x) = x$.

Example 5: Find $\dfrac{2}{5} \times \dfrac{-3}{7} - \dfrac{1}{14} - \dfrac{3}{7} \times \dfrac{3}{5}$

Solution: $\dfrac{2}{5} \times \dfrac{-3}{7} - \dfrac{1}{14} - \dfrac{3}{7} \times \dfrac{3}{5} = \dfrac{2}{5} \times \dfrac{-3}{7} - \dfrac{3}{7} \times \dfrac{3}{5} - \dfrac{1}{14}$ (by commutativity)

$$= \dfrac{2}{5} \times \dfrac{-3}{7} + \left(\dfrac{-3}{7}\right) \times \dfrac{3}{5} - \dfrac{1}{14}$$

$$= \dfrac{-3}{7}\left(\dfrac{2}{5} + \dfrac{3}{5}\right) - \dfrac{1}{14} \qquad \text{(by distributivity)}$$

$$= \dfrac{-3}{7} \times 1 - \dfrac{1}{14} = \dfrac{-6-1}{14} = \dfrac{-1}{2}$$

EXERCISE 1.1

1. Using appropriate properties find.

   (i) $-\dfrac{2}{3} \times \dfrac{3}{5} + \dfrac{5}{2} - \dfrac{3}{5} \times \dfrac{1}{6}$

   (ii) $\dfrac{2}{5} \times \left(-\dfrac{3}{7}\right) - \dfrac{1}{6} \times \dfrac{3}{2} + \dfrac{1}{14} \times \dfrac{2}{5}$

2. Write the additive inverse of each of the following.

   (i) $\dfrac{2}{8}$ (ii) $\dfrac{-5}{9}$ (iii) $\dfrac{-6}{-5}$ (iv) $\dfrac{2}{-9}$ (v) $\dfrac{19}{-6}$

3. Verify that $-(-x) = x$ for.

   (i) $x = \dfrac{11}{15}$ (ii) $x = -\dfrac{13}{17}$

4. Find the multiplicative inverse of the following.

   (i) -13 (ii) $\dfrac{-13}{19}$ (iii) $\dfrac{1}{5}$ (iv) $\dfrac{-5}{8} \times \dfrac{-3}{7}$

   (v) $-1 \times \dfrac{-2}{5}$ (vi) -1

5. Name the property under multiplication used in each of the following.

   (i) $\dfrac{-4}{5} \times 1 = 1 \times \dfrac{-4}{5} = -\dfrac{4}{5}$ (ii) $-\dfrac{13}{17} \times \dfrac{-2}{7} = \dfrac{-2}{7} \times \dfrac{-13}{17}$

   (iii) $\dfrac{-19}{29} \times \dfrac{29}{-19} = 1$

6. Multiply $\dfrac{6}{13}$ by the reciprocal of $\dfrac{-7}{16}$.

7. Tell what property allows you to compute $\dfrac{1}{3} \times \left(6 \times \dfrac{4}{3}\right)$ as $\left(\dfrac{1}{3} \times 6\right) \times \dfrac{4}{3}$.

8. Is $\dfrac{8}{9}$ the multiplicative inverse of $-1\dfrac{1}{8}$? Why or why not?

9. Is 0.3 the multiplicative inverse of $3\dfrac{1}{3}$? Why or why not?

10. Write.

   (i)   The rational number that does not have a reciprocal.

   (ii)   The rational numbers that are equal to their reciprocals.

   (iii)   The rational number that is equal to its negative.

11. Fill in the blanks.

   (i)   Zero has _____ reciprocal.

   (ii)   The numbers _____ and _____ are their own reciprocals

   (iii)   The reciprocal of – 5 is _____.

   (iv)   Reciprocal of $\dfrac{1}{x}$, where $x \neq 0$ is _____.

   (v)   The product of two rational numbers is always a _____.

   (vi)   The reciprocal of a positive rational number is _____.

1.3 Representation of Rational Numbers on the Number Line

You have learnt to represent natural numbers, whole numbers, integers and rational numbers on a number line. Let us revise them.

Natural numbers

  (i)
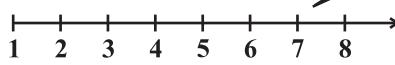
> The line extends indefinitely only to the right side of 1.

Whole numbers

  (ii)
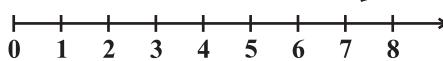
> The line extends indefinitely to the right, but from 0. There are no numbers to the left of 0.

Integers

  (iii)
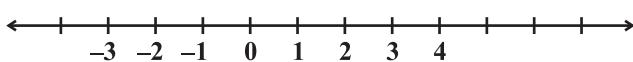
> The line extends indefinitely on both sides. Do you see any numbers between –1, 0; 0, 1 etc.?

Rational numbers

  (iv)
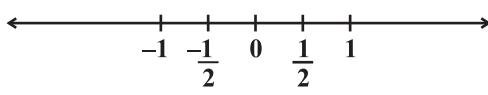
> The line extends indefinitely on both sides. But you can now see numbers between –1, 0; 0, 1 etc.

  (v)
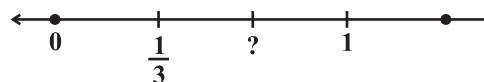
The point on the number line (iv) which is half way between 0 and 1 has been labelled $\dfrac{1}{2}$. Also, the first of the equally spaced points that divides the distance between 0 and 1 into three equal parts can be labelled $\dfrac{1}{3}$, as on number line (v). How would you label the second of these division points on number line (v)?

The point to be labelled is twice as far from and to the right of 0 as the point labelled $\frac{1}{3}$. So it is two times $\frac{1}{3}$, i.e., $\frac{2}{3}$. You can continue to label equally-spaced points on the number line in the same way. In this continuation, the next marking is 1. You can see that 1 is the same as $\frac{3}{3}$.

Then comes $\frac{4}{3}, \frac{5}{3}, \frac{6}{3}$ (or 2), $\frac{7}{3}$ and so on as shown on the number line (vi)

(vi)

$$\begin{array}{ccccccccccc} 0 & & & 1 & & & 2 & & & 3 & \\ \frac{0}{3} & \frac{1}{3} & \frac{2}{3} & \frac{3}{3} & \frac{4}{3} & \frac{5}{3} & \frac{6}{3} & \frac{7}{3} & \frac{8}{3} & \frac{9}{3} & \frac{10}{3} \end{array}$$
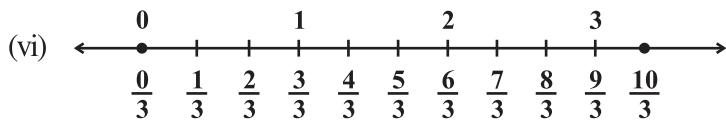
Similarly, to represent $\frac{1}{8}$, the number line may be divided into eight equal parts as shown:

$$\begin{array}{cc} 0 & 1 \end{array}$$
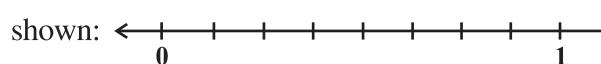
We use the number $\frac{1}{8}$ to name the first point of this division. The second point of division will be labelled $\frac{2}{8}$, the third point $\frac{3}{8}$, and so on as shown on number line (vii)

(vii)

$$\begin{array}{ccccccccc} & & & & & & & & 1 \\ 0 & \frac{1}{8} & \frac{2}{8} & \frac{3}{8} & \frac{4}{8} & \frac{5}{8} & \frac{6}{8} & \frac{7}{8} & \frac{8}{8} \end{array}$$
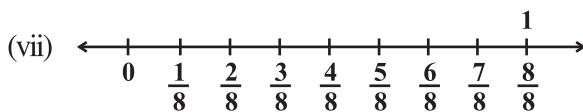
Any rational number can be represented on the number line in this way. In a rational number, the numeral below the bar, i.e., the denominator, tells the number of equal parts into which the first unit has been divided. The numeral above the bar i.e., the numerator, tells 'how many' of these parts are considered. So, a rational number such as $\frac{4}{9}$ means four of nine equal parts on the right of 0 (number line viii) and for $\frac{-7}{4}$, we make 7 markings of distance $\frac{1}{4}$ each on the *left* of zero and starting from 0. The seventh marking is $\frac{-7}{4}$ [number line (ix)].

(viii)

$$\begin{array}{cccccccccc} & & & & & & & & & 1 \\ 0 & \frac{1}{9} & \frac{2}{9} & \frac{3}{9} & \boxed{\frac{4}{9}} & \frac{5}{9} & \frac{6}{9} & \frac{7}{9} & \frac{8}{9} & \frac{9}{9} \end{array}$$

(ix)

$$\begin{array}{ccccccccc} -2 & & & & -1 & & & & \\ \frac{-8}{4} & \boxed{\frac{-7}{4}} & \frac{-6}{4} & \frac{-5}{4} & \frac{-4}{4} & \frac{-3}{4} & \frac{-2}{4} & \frac{-1}{4} & 0 \end{array}$$
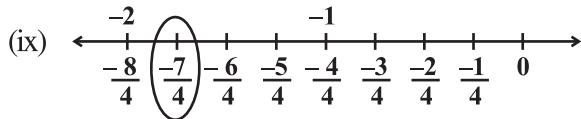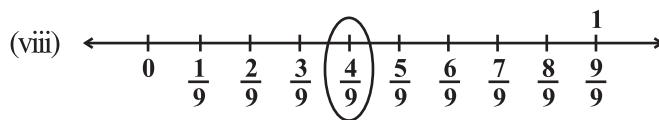
Write the rational number for each point labelled with a letter.

(i)

	A			B	C		D	E			
$\frac{0}{5}$	?	$\frac{2}{5}$	$\frac{3}{5}$	?	?	$\frac{6}{5}$	$\frac{7}{5}$	?	?	$\frac{10}{5}$	$\frac{11}{5}$ $\frac{12}{5}$

(ii)

	J		I	H		G		F		
$\frac{-12}{6}$	?	$\frac{-10}{6}$ $\frac{-9}{6}$	?	?	$\frac{-6}{6}$	?	$\frac{-4}{6}$ $\frac{-3}{6}$	?	$\frac{-1}{6}$	$\frac{0}{6}$
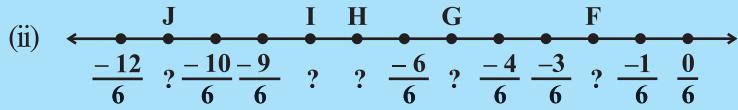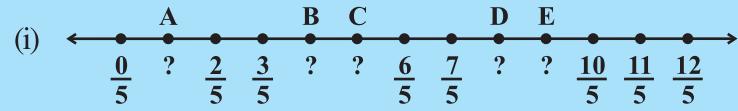
1.4 Rational Numbers between Two Rational Numbers

Can you tell the natural numbers between 1 and 5? They are 2, 3 and 4.

How many natural numbers are there between 7 and 9? There is one and it is 8.

How many natural numbers are there between 10 and 11? Obviously none.

List the integers that lie between –5 and 4. They are – 4, – 3, –2, –1, 0, 1, 2, 3.

How many integers are there between –1 and 1?

How many integers are there between –9 and –10?

You will find a definite number of natural numbers (integers) between two natural numbers (integers).

How many rational numbers are there between $\frac{3}{10}$ and $\frac{7}{10}$?

You may have thought that they are only $\frac{4}{10}, \frac{5}{10}$ and $\frac{6}{10}$.

But you can also write $\frac{3}{10}$ as $\frac{30}{100}$ and $\frac{7}{10}$ as $\frac{70}{100}$. Now the numbers, $\frac{31}{100}, \frac{32}{100}, \frac{33}{100}$

, ... $\frac{68}{100}, \frac{69}{100}$, are all between $\frac{3}{10}$ and $\frac{7}{10}$. The number of these rational numbers is 39.

Also $\frac{3}{10}$ can be expressed as $\frac{3000}{10000}$ and $\frac{7}{10}$ as $\frac{7000}{10000}$. Now, we see that the

rational numbers $\frac{3001}{10000}, \frac{3002}{10000}, ..., \frac{6998}{10000}, \frac{6999}{10000}$ are between $\frac{3}{10}$ and $\frac{7}{10}$. These are 3999 numbers in all.

In this way, we can go on inserting more and more rational numbers between $\frac{3}{10}$

and $\frac{7}{10}$. So unlike natural numbers and integers, the number of rational numbers between two rational numbers is not definite. Here is one more example.

How many rational numbers are there between $\frac{-1}{10}$ and $\frac{3}{10}$?

Obviously $\frac{0}{10}, \frac{1}{10}, \frac{2}{10}$ are rational numbers between the given numbers.

If we write $\dfrac{-1}{10}$ as $\dfrac{-10000}{100000}$ and $\dfrac{3}{10}$ as $\dfrac{30000}{100000}$, we get the rational numbers $\dfrac{-9999}{100000}, \dfrac{-9998}{100000}, ..., \dfrac{-29998}{100000}, \dfrac{29999}{100000}$, between $\dfrac{-1}{10}$ and $\dfrac{3}{10}$.

You will find that *you get countless rational numbers between any two given rational numbers*.

Example 6: Write any 3 rational numbers between –2 and 0.

Solution: –2 can be written as $\dfrac{-20}{10}$ and 0 as $\dfrac{0}{10}$.

Thus we have $\dfrac{-19}{10}, \dfrac{-18}{10}, \dfrac{-17}{10}, \dfrac{-16}{10}, \dfrac{-15}{10}, ..., \dfrac{-1}{10}$ between –2 and 0.

You can take any three of these.

Example 7: Find any ten rational numbers between $\dfrac{-5}{6}$ and $\dfrac{5}{8}$.

Solution: We first convert $\dfrac{-5}{6}$ and $\dfrac{5}{8}$ to rational numbers with the same denominators.

$$\dfrac{-5 \times 4}{6 \times 4} = \dfrac{-20}{24} \qquad \text{and} \qquad \dfrac{5 \times 3}{8 \times 3} = \dfrac{15}{24}$$

Thus we have $\dfrac{-19}{24}, \dfrac{-18}{24}, \dfrac{-17}{24}, ..., \dfrac{14}{24}$ as the rational numbers between $\dfrac{-20}{24}$ and $\dfrac{15}{24}$.

You can take any ten of these.

Another Method

Let us find rational numbers between 1 and 2. One of them is 1.5 or $1\dfrac{1}{2}$ or $\dfrac{3}{2}$. This is the **mean** of 1 and 2. You have studied mean in Class VII.

We find that *between any two given numbers, we need not necessarily get an integer but there will always lie a rational number*.

We can use the idea of mean also to find rational numbers between any two given rational numbers.

Example 8: Find a rational number between $\dfrac{1}{4}$ and $\dfrac{1}{2}$.

Solution: We find the mean of the given rational numbers.

$$\left(\dfrac{1}{4} + \dfrac{1}{2}\right) \div 2 = \left(\dfrac{1+2}{4}\right) \div 2 = \dfrac{3}{4} \times \dfrac{1}{2} = \dfrac{3}{8}$$

$\dfrac{3}{8}$ lies between $\dfrac{1}{4}$ and $\dfrac{1}{2}$.

This can be seen on the number line also.
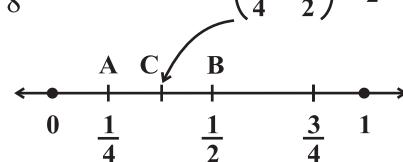
We find the mid point of AB which is C, represented by $\left(\dfrac{1}{4} + \dfrac{1}{2}\right) \div 2 = \dfrac{3}{8}$.

We find that $\dfrac{1}{4} < \dfrac{3}{8} < \dfrac{1}{2}$.

If a and b are two rational numbers, then $\dfrac{a+b}{2}$ is a rational number between a and b such that $a < \dfrac{a+b}{2} < b$.

This again shows that there are countless number of rational numbers between any two given rational numbers.

Example 9: Find three rational numbers between $\dfrac{1}{4}$ and $\dfrac{1}{2}$.

Solution: We find the mean of the given rational numbers.

As given in the above example, the mean is $\dfrac{3}{8}$ and $\dfrac{1}{4} < \dfrac{3}{8} < \dfrac{1}{2}$.
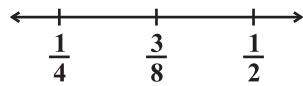
We now find another rational number between $\dfrac{1}{4}$ and $\dfrac{3}{8}$. For this, we again find the mean of $\dfrac{1}{4}$ and $\dfrac{3}{8}$. That is, $\left(\dfrac{1}{4} + \dfrac{3}{8}\right) \div 2 = \dfrac{5}{8} \times \dfrac{1}{2} = \dfrac{5}{16}$

$$\dfrac{1}{4} < \dfrac{5}{16} < \dfrac{3}{8} < \dfrac{1}{2}$$
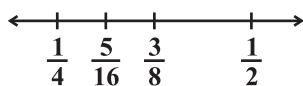
Now find the mean of $\dfrac{3}{8}$ and $\dfrac{1}{2}$. We have, $\left(\dfrac{3}{8} + \dfrac{1}{2}\right) \div 2 = \dfrac{7}{8} \times \dfrac{1}{2} = \dfrac{7}{16}$

Thus we get $\dfrac{1}{4} < \dfrac{5}{16} < \dfrac{3}{8} < \dfrac{7}{16} < \dfrac{1}{2}$.
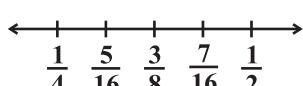
Thus, $\dfrac{5}{16}, \dfrac{3}{8}, \dfrac{7}{16}$ are the three rational numbers between $\dfrac{1}{4}$ and $\dfrac{1}{2}$.

This can clearly be shown on the number line as follows:

$$\left(\dfrac{1}{4} + \dfrac{3}{8}\right) \div 2 = \dfrac{5}{16} \qquad \dfrac{7}{16} = \left(\dfrac{3}{8} + \dfrac{1}{2}\right) \div 2$$
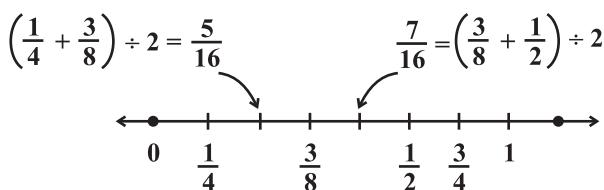
In the same way we can obtain as many rational numbers as we want between two given rational numbers . You have noticed that there are countless rational numbers between any two given rational numbers.

EXERCISE 1.2

1. Represent these numbers on the number line. (i) $\dfrac{7}{4}$ (ii) $\dfrac{-5}{6}$

2. Represent $\dfrac{-2}{11}, \dfrac{-5}{11}, \dfrac{-9}{11}$ on the number line.

3. Write five rational numbers which are smaller than 2.

4. Find ten rational numbers between $\dfrac{-2}{5}$ and $\dfrac{1}{2}$.

5. Find five rational numbers between.

   (i) $\dfrac{2}{3}$ and $\dfrac{4}{5}$ (ii) $\dfrac{-3}{2}$ and $\dfrac{5}{3}$ (iii) $\dfrac{1}{4}$ and $\dfrac{1}{2}$

6. Write five rational numbers greater than –2.

7. Find ten rational numbers between $\dfrac{3}{5}$ and $\dfrac{3}{4}$.

WHAT HAVE WE DISCUSSED?

1. Rational numbers are **closed** under the operations of addition, subtraction and multiplication.
2. The operations addition and multiplication are
   (i) **commutative** for rational numbers.
   (ii) **associative** for rational numbers.
3. The rational number 0 is the **additive identity** for rational numbers.
4. The rational number 1 is the **multiplicative identity** for rational numbers.
5. The **additive inverse** of the rational number $\dfrac{a}{b}$ is $-\dfrac{a}{b}$ and vice-versa.
6. The **reciprocal** or **multiplicative inverse** of the rational number $\dfrac{a}{b}$ is $\dfrac{c}{d}$ if $\dfrac{a}{b} \times \dfrac{c}{d} = 1$.
7. **Distributivity** of rational numbers: For all rational numbers a, b and c,
   $a(b + c) = ab + ac$ and $a(b - c) = ab - ac$
8. Rational numbers can be represented on a number line.
9. Between any two given rational numbers there are countless rational numbers. The idea of **mean** helps us to find rational numbers between two rational numbers.

Linear Equations in One Variable

0852CH02

2.1 Introduction

In the earlier classes, you have come across several **algebraic expressions** and **equations**.
Some examples of expressions we have so far worked with are:

$$5x,\ 2x - 3,\ 3x + y,\ 2xy + 5,\ xyz + x + y + z,\ x^2 + 1,\ y + y^2$$

Some examples of equations are: $5\,x = 25,\ 2\,x - 3 = 9,\ 2y + \dfrac{5}{2} = \dfrac{37}{2},\ 6z + 10 = -2$

You would remember that equations use the *equality* (=) sign; it is missing in expressions.

Of these given expressions, many have more than one variable. For example, $2xy + 5$ has two variables. We however, restrict to expressions with only one variable when we form equations. Moreover, the expressions we use to form equations are linear. This means that the highest power of the variable appearing in the expression is 1.

These are linear expressions:

$$2x,\ 2x + 1,\ 3y - 7,\ 12 - 5z,\ \frac{5}{4}(x - 4) + 10$$

These are **not** linear expressions:

$x^2 + 1,\ y + y^2,\ 1 + z + z^2 + z^3$ (since highest power of variable > 1)

Here we will deal with equations with linear expressions in one variable only. Such equations are known as **linear equations in one variable**. The simple equations which you studied in the earlier classes were all of this type.

Let us briefly revise what we know:

(a) *An algebraic equation is an equality involving variables*. It has an *equality sign*. The expression on the left of the equality sign is the *Left Hand Side* (LHS). The expression on the right of the equality sign is the *Right Hand Side* (RHS).

the variable   equality

equation

$$2x - 3 = 7$$

$$2x - 3 = \text{LHS}$$
$$7 = \text{RHS}$$
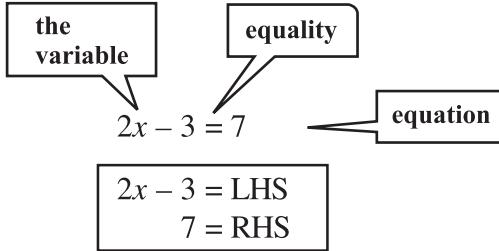
(b) In an equation the *values of the expressions on the LHS and RHS are equal*. This happens to be *true* only for certain values of the variable. These values are the **solutions** of the equation.

> $x = 5$ is the solution of the equation $2x - 3 = 7$. For $x = 5$,
> LHS $= 2 \times 5 - 3 = 7 =$ RHS
> On the other hand $x = 10$ is not a solution of the equation. For $x = 10$, LHS $= 2 \times 10 - 3 = 17$. This is not equal to the RHS

(c) *How to find the solution of an equation?*

We assume that the two sides of the equation are balanced. We perform the same mathematical operations on both sides of the equation, so that the balance is not disturbed. A few such steps give the solution.

2.2 Solving Equations which have Linear Expressions on one Side and Numbers on the other Side

Let us recall the technique of solving equations with some examples. Observe the solutions; they can be any rational number.

Example 1: Find the solution of $2x - 3 = 7$

Solution:

Step 1 Add 3 to both sides.

$$2x - 3 + 3 = 7 + 3 \qquad \text{(The balance is not disturbed)}$$

or $\qquad 2x = 10$

Step 2 Next divide both sides by 2.

$$\frac{2x}{2} = \frac{10}{2}$$

or $\qquad x = 5 \qquad$ (required solution)

Example 2: Solve $2y + 9 = 4$

Solution: Transposing 9 to RHS

$$2y = 4 - 9$$

or $\qquad 2y = -5$

Dividing both sides by 2, $\qquad y = \dfrac{-5}{2} \qquad$ (solution)

To check the answer: LHS $= 2\left(\dfrac{-5}{2}\right) + 9 = -5 + 9 = 4 =$ RHS $\qquad$ (as required)

Do you notice that the solution $\left(\dfrac{-5}{2}\right)$ is a rational number? In Class VII, the equations we solved did not have such solutions.

Example 3: Solve $\dfrac{x}{3} + \dfrac{5}{2} = -\dfrac{3}{2}$

Solution: Transposing $\dfrac{5}{2}$ to the RHS, we get $\dfrac{x}{3} = \dfrac{-3}{2} - \dfrac{5}{2} = -\dfrac{8}{2}$

or $\qquad\qquad\qquad\qquad\qquad \dfrac{x}{3} = -4$

Multiply both sides by 3, $\qquad\quad x = -4 \times 3$

or $\qquad\qquad\qquad\qquad\qquad x = -12 \qquad\qquad\qquad\qquad\qquad$ (solution)

Check: LHS $= -\dfrac{12}{3} + \dfrac{5}{2} = -4 + \dfrac{5}{2} = \dfrac{-8+5}{2} = \dfrac{-3}{2} = $ RHS $\qquad\qquad$ (as required)

Do you now see that the coefficient of a variable in an equation need not be an integer?

Example 4: Solve $\dfrac{15}{4} - 7x = 9$

Solution: We have $\qquad\qquad\qquad \dfrac{15}{4} - 7x = 9$

or $\qquad\qquad\qquad\qquad -7x = 9 - \dfrac{15}{4} \qquad$ (transposing $\dfrac{15}{4}$ to R H S)

or $\qquad\qquad\qquad\qquad -7x = \dfrac{21}{4}$

or $\qquad\qquad\qquad\qquad\quad x = \dfrac{21}{4 \times (-7)} \qquad$ (dividing both sides by -7)

or $\qquad\qquad\qquad\qquad\quad x = -\dfrac{3 \times 7}{4 \times 7}$

or $\qquad\qquad\qquad\qquad\quad x = -\dfrac{3}{4} \qquad\qquad\qquad\qquad\qquad$ (solution)

Check: LHS $= \dfrac{15}{4} - 7\left(\dfrac{-3}{4}\right) = \dfrac{15}{4} = \dfrac{21}{4} + \dfrac{36}{4} = 9 = $ RHS $\qquad\qquad$ (as required)

EXERCISE 2.1

Solve the following equations.

1. $x - 2 = 7$ 　　　　　　　 **2.** $y + 3 = 10$ 　　　　　　 **3.** $6 = z + 2$

4. $\dfrac{3}{7} + x = \dfrac{17}{7}$ 　　　　 **5.** $6x = 12$ 　　　　　　　 **6.** $\dfrac{t}{5} = 10$

7. $\dfrac{2x}{3} = 18$ 　　　　　　 **8.** $1.6 = \dfrac{y}{1.5}$ 　　　　　 **9.** $7x - 9 = 16$

10. $14y - 8 = 13$ **11.** $17 + 6p = 9$ **12.** $\dfrac{x}{3} + 1 = \dfrac{7}{15}$

2.3 Some Applications

We begin with a simple example.

Sum of two numbers is 74. One of the numbers is 10 more than the other. What are the numbers?

We have a puzzle here. We do not know either of the two numbers, and we have to find them. We are given two conditions.

(i) One of the numbers is 10 more than the other.

(ii) Their sum is 74.

We already know from Class VII how to proceed. If the smaller number is taken to be x, the larger number is 10 more than x, i.e., $x + 10$. The other condition says that the sum of these two numbers x and $x + 10$ is 74.

This means that $x + (x + 10) = 74$.

or $2x + 10 = 74$

Transposing 10 to RHS, $2x = 74 - 10$

or $2x = 64$

Dividing both sides by 2, $x = 32.$ This is one number.

The other number is $x + 10 = 32 + 10 = 42$

The desired numbers are 32 and 42. (Their sum is indeed 74 as given and also one number is 10 more than the other.)

We shall now consider several examples to show how useful this method is.

Example 5: What should be added to twice the rational number $\dfrac{-7}{3}$ to get $\dfrac{3}{7}$?

Solution: Twice the rational number $\dfrac{-7}{3}$ is $2 \times \left(\dfrac{-7}{3}\right) = \dfrac{-14}{3}$. Suppose x added to this number gives $\dfrac{3}{7}$; i.e.,

$$x + \left(\dfrac{-14}{3}\right) = \dfrac{3}{7}$$

or $x - \dfrac{14}{3} = \dfrac{3}{7}$

or $x = \dfrac{3}{7} + \dfrac{14}{3}$ (transposing $\dfrac{14}{3}$ to RHS)

$$= \dfrac{(3 \times 3) + (14 \times 7)}{21} = \dfrac{9 + 98}{21} = \dfrac{107}{21}.$$

Thus $\dfrac{107}{21}$ should be added to $2 \times \left(\dfrac{-7}{3}\right)$ to give $\dfrac{3}{7}$.

Example 6: The perimeter of a rectangle is 13 cm and its width is $2\dfrac{3}{4}$ cm. Find its length.

Solution: Assume the length of the rectangle to be x cm.

The perimeter of the rectangle $= 2 \times$ (length + width)

$$= 2 \times (x + 2\dfrac{3}{4})$$

$$= 2\left(x + \dfrac{11}{4}\right)$$

The perimeter is given to be 13 cm. Therefore,

$$2\left(x + \dfrac{11}{4}\right) = 13$$

or $\qquad x + \dfrac{11}{4} = \dfrac{13}{2}$ (dividing both sides by 2)

or $\qquad x = \dfrac{13}{2} - \dfrac{11}{4}$

$$= \dfrac{26}{4} - \dfrac{11}{4} = \dfrac{15}{4} = 3\dfrac{3}{4}$$

The length of the rectangle is $3\dfrac{3}{4}$ cm.

Example 7: The present age of Sahil's mother is three times the present age of Sahil. After 5 years their ages will add to 66 years. Find their present ages.

Solution: Let Sahil's present age be x years.

We could also choose Sahil's age 5 years later to be x and proceed. Why don't you try it that way?

	Sahil	Mother	Sum
Present age	x	$3x$	
Age 5 years later	$x + 5$	$3x + 5$	$4x + 10$

It is given that this sum is 66 years.

Therefore, $\qquad 4x + 10 = 66$

This equation determines Sahil's present age which is x years. To solve the equation,

we transpose 10 to RHS,

$$4x = 66 - 10$$

or $$4x = 56$$

or $$x = \frac{56}{4} = 14 \qquad \text{(solution)}$$

Thus, Sahil's present age is 14 years and his mother's age is 42 years. (You may easily check that 5 years from now the sum of their ages will be 66 years.)

Example 8: Bansi has 3 times as many two-rupee coins as he has five-rupee coins. If he has in all a sum of ₹ 77, how many coins of each denomination does he have?

Solution: Let the number of five-rupee coins that Bansi has be x. Then the number of two-rupee coins he has is 3 times x or $3x$.

The amount Bansi has:

  (i)   from 5 rupee coins, ₹ $5 \times x$ = ₹ $5x$

 (ii)   from 2 rupee coins, ₹ $2 \times 3x$ = ₹ $6x$

        Hence the total money he has = ₹ $11x$

But this is given to be ₹ 77; therefore,

$$11x = 77$$

or $$x = \frac{77}{11} = 7$$

Thus,   number of five-rupee coins = x = 7

and   number of two-rupee coins = $3x$ = 21   (solution)
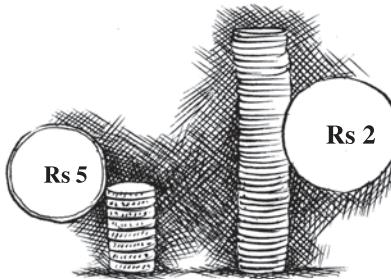
(You can check that the total money with Bansi is ₹ 77.)

Example 9: The sum of three consecutive multiples of 11 is 363. Find these multiples.

Solution: If x is a multiple of 11, the next multiple is $x + 11$. The next to this is $x + 11 + 11$ or $x + 22$. So we can take three consecutive multiples of 11 as x, $x + 11$ and $x + 22$.

It is given that the sum of these consecutive multiples of 11 is 363. This will give the following equation:

$$x + (x + 11) + (x + 22) = 363$$

or $$x + x + 11 + x + 22 = 363$$

or $$3x + 33 = 363$$

or $$3x = 363 - 33$$

or $$3x = 330$$

> Alternatively, we may think of the multiple of 11 immediately before x. This is $(x - 11)$. Therefore, we may take three consecutive multiples of 11 as $x - 11$, x, $x + 11$.
>
> In this case we arrive at the equation
> $$(x - 11) + x + (x + 11) = 363$$
> or $$3x = 363$$

or $\qquad x = \dfrac{330}{3}$

$\qquad\qquad = 110$

Hence, the three consecutive multiples are 110, 121, 132 (answer).

We can see that we can adopt different ways to find a solution for the problem.

Example 10: The difference between two whole numbers is 66. The ratio of the two numbers is 2 : 5. What are the two numbers?

Solution: Since the ratio of the two numbers is 2 : 5, we may take one number to be $2x$ and the other to be $5x$ (Note that $2x : 5x$ is same as 2 : 5.)

The difference between the two numbers is $(5x - 2x)$. It is given that the difference is 66. Therefore,

$$5x - 2x = 66$$

or $\qquad\qquad 3x = 66$

or $\qquad\qquad x = 22$

Since the numbers are $2x$ and $5x$, they are 2×22 or 44 and 5×22 or 110, respectively.

The difference between the two numbers is $110 - 44 = 66$ as desired.

Example 11: Deveshi has a total of ₹ 590 as currency notes in the denominations of ₹ 50, ₹ 20 and ₹ 10. The ratio of the number of ₹ 50 notes and ₹ 20 notes is 3:5. If she has a total of 25 notes, how many notes of each denomination she has?

Solution: Let the number of ₹ 50 notes and ₹ 20 notes be $3x$ and $5x$, respectively.

But she has 25 notes in total.

Therefore, the number of ₹ 10 notes = $25 - (3x + 5x) = 25 - 8x$

The amount she has

from ₹ 50 notes : $3x \times 50 = ₹ 150x$

from ₹ 20 notes : $5x \times 20 = ₹ 100x$

from ₹ 10 notes : $(25 - 8x) \times 10 = ₹ (250 - 80x)$

   Hence the total money she has $= 150x + 100x + (250 - 80x) = ₹ (170x + 250)$

But she has ₹ 590. Therefore, $\qquad 170x + 250 = 590$

   or $\qquad\qquad 170x = 590 - 250 = 340$

   or $\qquad\qquad x = \dfrac{340}{170} = 2$

$\qquad$ The number of ₹ 50 notes she has $= 3x$

$\qquad\qquad\qquad\qquad = 3 \times 2 = 6$

$\qquad$ The number of ₹ 20 notes she has $= 5x = 5 \times 2 = 10$

$\qquad$ The number of ₹ 10 notes she has $= 25 - 8x$

$\qquad\qquad\qquad\qquad = 25 - (8 \times 2) = 25 - 16 = 9$

▲ **EXERCISE 2.2**

1. If you subtract $\frac{1}{2}$ from a number and multiply the result by $\frac{1}{2}$, you get $\frac{1}{8}$. What is the number?

2. The perimeter of a rectangular swimming pool is 154 m. Its length is 2 m more than twice its breadth. What are the length and the breadth of the pool?

3. The base of an isosceles triangle is $\frac{4}{3}$ cm. The perimeter of the triangle is $4\frac{2}{15}$ cm. What is the length of either of the remaining equal sides?

4. Sum of two numbers is 95. If one exceeds the other by 15, find the numbers.

5. Two numbers are in the ratio 5:3. If they differ by 18, what are the numbers?

6. Three consecutive integers add up to 51. What are these integers?

7. The sum of three consecutive multiples of 8 is 888. Find the multiples.

8. Three consecutive integers are such that when they are taken in increasing order and multiplied by 2, 3 and 4 respectively, they add up to 74. Find these numbers.

9. The ages of Rahul and Haroon are in the ratio 5:7. Four years later the sum of their ages will be 56 years. What are their present ages?

10. The number of boys and girls in a class are in the ratio 7:5. The number of boys is 8 more than the number of girls. What is the total class strength?

11. Baichung's father is 26 years younger than Baichung's grandfather and 29 years older than Baichung. The sum of the ages of all the three is 135 years. What is the age of each one of them?

12. Fifteen years from now Ravi's age will be four times his present age. What is Ravi's present age?

13. A rational number is such that when you multiply it by $\frac{5}{2}$ and add $\frac{2}{3}$ to the product, you get $-\frac{7}{12}$. What is the number?

14. Lakshmi is a cashier in a bank. She has currency notes of denominations ₹ 100, ₹ 50 and ₹ 10, respectively. The ratio of the number of these notes is 2:3:5. The total cash with Lakshmi is ₹ 4,00,000. How many notes of each denomination does she have?

15. I have a total of ₹ 300 in coins of denomination ₹ 1, ₹ 2 and ₹ 5. The number of ₹ 2 coins is 3 times the number of ₹ 5 coins. The total number of coins is 160. How many coins of each denomination are with me?

16. The organisers of an essay competition decide that a winner in the competition gets a prize of ₹ 100 and a participant who does not win gets a prize of ₹ 25. The total prize money distributed is ₹ 3,000. Find the number of winners, if the total number of participants is 63.

2.4 Solving Equations having the Variable on both Sides

An equation is the equality of the values of two expressions. In the equation $2x - 3 = 7$, the two expressions are $2x - 3$ and 7. In most examples that we have come across so far, the RHS is just a number. But this need not always be so; both sides could have expressions with variables. For example, the equation $2x - 3 = x + 2$ has expressions with a variable on both sides; the expression on the LHS is $(2x - 3)$ and the expression on the RHS is $(x + 2)$.

- We now discuss how to solve such equations which have expressions with the variable on both sides.

Example 12: Solve $2x - 3 = x + 2$

Solution: We have

$$2x = x + 2 + 3$$
or $$2x = x + 5$$
or $$2x - x = x + 5 - x \quad \text{(subtracting } x \text{ from both sides)}$$
or $$x = 5 \quad \text{(solution)}$$

Here we subtracted from both sides of the equation, not a number (constant), but a term involving the variable. We can do this as variables are also numbers. Also, note that subtracting x from both sides amounts to transposing x to LHS.

Example 13: Solve $5x + \dfrac{7}{2} = \dfrac{3}{2}x - 14$

Solution: Multiply both sides of the equation by 2. We get

$$2 \times \left(5x + \frac{7}{2}\right) = 2 \times \left(\frac{3}{2}x - 14\right)$$

$$(2 \times 5x) + \left(2 \times \frac{7}{2}\right) = \left(2 \times \frac{3}{2}x\right) - (2 \times 14)$$

or $$10x + 7 = 3x - 28$$

or $$10x - 3x + 7 = -28 \quad \text{(transposing } 3x \text{ to LHS)}$$

or $$7x + 7 = -28$$

or $$7x = -28 - 7$$

or $$7x = -35$$

or $$x = \frac{-35}{7} \qquad \text{or} \qquad x = -5 \qquad \text{(solution)}$$

EXERCISE 2.3

Solve the following equations and check your results.

1. $3x = 2x + 18$ **2.** $5t - 3 = 3t - 5$ **3.** $5x + 9 = 5 + 3x$

4. $4z + 3 = 6 + 2z$ **5.** $2x - 1 = 14 - x$ **6.** $8x + 4 = 3(x - 1) + 7$

7. $x = \dfrac{4}{5}(x + 10)$ **8.** $\dfrac{2x}{3} + 1 = \dfrac{7x}{15} + 3$ **9.** $2y + \dfrac{5}{3} = \dfrac{26}{3} - y$

10. $3m = 5\,m - \dfrac{8}{5}$

2.5 Some More Applications

Example 14: The digits of a two-digit number differ by 3. If the digits are interchanged, and the resulting number is added to the original number, we get 143. What can be the original number?

Solution: Take, for example, a two-digit number, say, 56. It can be written as $56 = (10 \times 5) + 6$.

If the digits in 56 are interchanged, we get 65, which can be written as $(10 \times 6) + 5$.

Let us take the two digit number such that the digit in the units place is b. The digit in the tens place differs from b by 3. Let us take it as $b + 3$. So the two-digit number is $10(b + 3) + b = 10b + 30 + b = 11b + 30$.

With interchange of digits, the resulting two-digit number will be

$$10b + (b + 3) = 11b + 3$$

> Could we take the tens place digit to be $(b - 3)$? Try it and see what solution you get.

If we add these two two-digit numbers, their sum is

$$(11b + 30) + (11b + 3) = 11b + 11b + 30 + 3 = 22b + 33$$

It is given that the sum is 143. Therefore, $22b + 33 = 143$

or $22b = 143 - 33$

or $22b = 110$

> Remember, this is the solution when we choose the tens digits to be 3 more than the unit's digits. What happens if we take the tens digit to be $(b - 3)$?

or $b = \dfrac{110}{22}$

or $b = 5$

The units digit is 5 and therefore the tens digit is $5 + 3$ which is 8. The number is 85.

> The statement of the example is valid for both 58 and 85 and both are correct answers.

Check: On interchange of digits the number we get is 58. The sum of 85 and 58 is 143 as given.

Example 15: Arjun is twice as old as Shriya. Five years ago his age was three times Shriya's age. Find their present ages.

Solution: Let us take Shriya's present age to be x years.

Then Arjun's present age would be $2x$ years.

Shriya's age five years ago was $(x-5)$ years.

Arjun's age five years ago was $(2x-5)$ years.

It is given that Arjun's age five years ago was three times Shriya's age.

Thus, $\qquad\qquad 2x - 5 = 3(x-5)$

or $\qquad\qquad\qquad 2x - 5 = 3x - 15$

or $\qquad\qquad\qquad 15 - 5 = 3x - 2x$

or $\qquad\qquad\qquad\qquad 10 = x$

So, Shriya's present age $= x = 10$ years.

Therefore, Arjun's present age $= 2\ x = 2 \times 10 = 20$ years.

EXERCISE 2.4

1. Amina thinks of a number and subtracts $\dfrac{5}{2}$ from it. She multiplies the result by 8. The result now obtained is 3 times the same number she thought of. What is the number?

2. A positive number is 5 times another number. If 21 is added to both the numbers, then one of the new numbers becomes twice the other new number. What are the numbers?

3. Sum of the digits of a two-digit number is 9. When we interchange the digits, it is found that the resulting new number is greater than the original number by 27. What is the two-digit number?

4. One of the two digits of a two digit number is three times the other digit. If you interchange the digits of this two-digit number and add the resulting number to the original number, you get 88. What is the original number?

5. Shobo's mother's present age is six times Shobo's present age. Shobo's age five years from now will be one third of his mother's present age. What are their present ages?

6. There is a narrow rectangular plot, reserved for a school, in Mahuli village. The length and breadth of the plot are in the ratio 11:4. At the rate ₹100 per metre it will cost the village panchayat ₹ 75000 to fence the plot. What are the dimensions of the plot?

7. Hasan buys two kinds of cloth materials for school uniforms, shirt material that costs him ₹ 50 per metre and trouser material that costs him ₹ 90 per metre.

For every 3 meters of the shirt material he buys 2 metres of the trouser material. He sells the materials at 12% and 10% profit respectively. His total sale is ₹ 36,600. How much trouser material did he buy?

8. Half of a herd of deer are grazing in the field and three fourths of the remaining are playing nearby. The rest 9 are drinking water from the pond. Find the number of deer in the herd.

9. A grandfather is ten times older than his granddaughter. He is also 54 years older than her. Find their present ages.

10. Aman's age is three times his son's age. Ten years ago he was five times his son's age. Find their present ages.

2.6 Reducing Equations to Simpler Form

Example 16: Solve $\dfrac{6x+1}{3}+1=\dfrac{x-3}{6}$

Solution: Multiplying both sides of the equation by 6,

> Why 6? Because it is the smallest multiple (or LCM) of the given denominators.

$$\frac{6\,(6x+1)}{3}+6\times 1 = \frac{6(x-3)}{6}$$

or $\qquad\qquad 2\,(6x+1)+6 = x-3$

or $\qquad\qquad 12x+2+6 = x-3$ $\qquad\qquad$ (opening the brackets)

or $\qquad\qquad 12x+8 = x-3$

or $\qquad\qquad 12x-x+8 = -3$

or $\qquad\qquad 11x+8 = -3$

or $\qquad\qquad 11x = -3-8$

or $\qquad\qquad 11x = -11$

or $\qquad\qquad x = -1$ $\qquad\qquad$ (required solution)

Check: LHS $=\dfrac{6(-1)+1}{3}+1 \quad \dfrac{-6+1}{3}+1=1 \;=\; \dfrac{-5}{3}=\dfrac{3}{3} \quad \dfrac{-5+3}{3}=\dfrac{-2}{3}$

$\qquad\qquad$ RHS $= \dfrac{(-1)-3}{6} = \dfrac{-4}{6} = \dfrac{-2}{3}$

$\qquad\qquad$ LHS = RHS. $\qquad$ (as required)

Example 17: Solve $5x-2\,(2x-7) = 2\,(3x-1)+\dfrac{7}{2}$

Solution: Let us open the brackets,

$\qquad\qquad$ LHS $= 5x-4x+14 = x+14$

$$\text{RHS} = 6x - 2 + \frac{7}{2} = 6x - \frac{4}{2} + \frac{7}{2} = 6x + \frac{3}{2}$$

The equation is $x + 14 = 6x + \dfrac{3}{2}$

or $\qquad\qquad 14 = 6x - x + \dfrac{3}{2}$

or $\qquad\qquad 14 = 5x + \dfrac{3}{2}$

or $\qquad\qquad 14 - \dfrac{3}{2} = 5x \qquad\qquad$ (transposing $\dfrac{3}{2}$)

or $\qquad\qquad \dfrac{28 - 3}{2} = 5x$

or $\qquad\qquad \dfrac{25}{2} = 5x$

or $\qquad\qquad x = \dfrac{25}{2} \times \dfrac{1}{5} = \dfrac{5 \times 5}{2 \times 5} = \dfrac{5}{2}$

> Did you observe how we simplified the form of the given equation? Here, we had to multiply both sides of the equation by the LCM of the denominators of the terms in the expressions of the equation.

Therefore, required solution is $x = \dfrac{5}{2}$.

Check: LHS $= 5 \times \dfrac{5}{2} - 2\left(\dfrac{5}{2} \times 2 - 7\right)$

$$= \frac{25}{2} - 2(5 - 7) = \frac{25}{2} - 2(-2) = \frac{25}{2} + 4 = \frac{25 + 8}{2} = \frac{33}{2}$$

$$\text{RHS} = 2\left(\frac{5}{2} \times 3 - 1\right) + \frac{7}{2} = 2\left(\frac{15}{2} - \frac{2}{2}\right) + \frac{7}{2} = \frac{2 \times 13}{2} + \frac{7}{2}$$

$$= \frac{26 + 7}{2} = \frac{33}{2} = \text{LHS.} \quad \text{(as required)}$$

> Note, in this example we brought the equation to a simpler form by opening brackets and combining like terms on both sides of the equation.

EXERCISE 2.5

Solve the following linear equations.

1. $\dfrac{x}{2} - \dfrac{1}{5} = \dfrac{x}{3} + \dfrac{1}{4}$

2. $\dfrac{n}{2} - \dfrac{3n}{4} + \dfrac{5n}{6} = 21$

3. $x + 7 - \dfrac{8x}{3} = \dfrac{17}{6} - \dfrac{5x}{2}$

4. $\dfrac{x-5}{3} = \dfrac{x-3}{5}$ **5.** $\dfrac{3t-2}{4} - \dfrac{2t+3}{3} = \dfrac{2}{3} - t$ **6.** $m - \dfrac{m-1}{2} = 1 - \dfrac{m-2}{3}$

Simplify and solve the following linear equations.

7. $3(t-3) = 5(2t+1)$ **8.** $15(y-4) - 2(y-9) + 5(y+6) = 0$

9. $3(5z-7) - 2(9z-11) = 4(8z-13) - 17$

10. $0.25(4f-3) = 0.05(10f-9)$

2.7 Equations Reducible to the Linear Form

Example 18: Solve $\dfrac{x+1}{2x+3} = \dfrac{3}{8}$

Solution: Observe that the equation is not a linear equation, since the expression on its LHS is not linear. But we can put it into the form of a linear equation. We multiply both sides of the equation by $(2x + 3)$,

$$\left(\dfrac{x+1}{2x+3}\right) \times (2x+3) = \dfrac{3}{8} \times (2x+3)$$

> Note that $2x + 3 \neq 0$ (Why?)

Notice that $(2x + 3)$ gets cancelled on the LHS We have then,

$$x + 1 = \dfrac{3(2x+3)}{8}$$

We have now a linear equation which we know how to solve.
Multiplying both sides by 8

$$8(x+1) = 3(2x+3)$$

> This step can be directly obtained by 'cross-multiplication'
>
> $\dfrac{x+1}{2x+3} \diagdown \dfrac{3}{8}$

or $8x + 8 = 6x + 9$

or $8x = 6x + 9 - 8$

or $8x = 6x + 1$

or $8x - 6x = 1$

or $2x = 1$

or $x = \dfrac{1}{2}$

The solution is $x = \dfrac{1}{2}$.

Check : Numerator of LHS $= \dfrac{1}{2} + 1 = \dfrac{1+2}{2} = \dfrac{3}{2}$

Denominator of LHS $= 2x + 3 = 2 \times \dfrac{1}{2} + 3 = 1 + 3 = 4$

LHS = numerator ÷ denominator = $\dfrac{3}{2} \div 4 = \dfrac{3}{2} \times \dfrac{1}{4} = \dfrac{3}{8}$

LHS = RHS.

Example 19: Present ages of Anu and Raj are in the ratio 4:5. Eight years from now the ratio of their ages will be 5:6. Find their present ages.

Solution: Let the present ages of Anu and Raj be $4x$ years and $5x$ years respectively.

After eight years. Anu's age = $(4x + 8)$ years;

After eight years, Raj's age = $(5x + 8)$ years.

Therefore, the ratio of their ages after eight years = $\dfrac{4x + 8}{5x + 8}$

This is given to be 5 : 6

Therefore, $\dfrac{4x + 8}{5x + 8} = \dfrac{5}{6}$

Cross-multiplication gives $6 (4x + 8) = 5 (5x + 8)$

or $24x + 48 = 25x + 40$

or $24x + 48 - 40 = 25x$

or $24x + 8 = 25x$

or $8 = 25x - 24x$

or $8 = x$

Therefore,   Anu's present age = $4x = 4 \times 8 = 32$ years

                Raj's present age = $5x = 5 \times 8 = 40$ years

EXERCISE 2.6

Solve the following equations.

1. $\dfrac{8x - 3}{3x} = 2$

2. $\dfrac{9x}{7 - 6x} = 15$

3. $\dfrac{z}{z + 15} = \dfrac{4}{9}$

4. $\dfrac{3y + 4}{2 - 6y} = \dfrac{-2}{5}$

5. $\dfrac{7y + 4}{y + 2} = \dfrac{-4}{3}$

6. The ages of Hari and Harry are in the ratio 5:7. Four years from now the ratio of their ages will be 3:4. Find their present ages.

7. The denominator of a rational number is greater than its numerator by 8. If the numerator is increased by 17 and the denominator is decreased by 1, the number obtained is $\dfrac{3}{2}$. Find the rational number.

WHAT HAVE WE DISCUSSED?

1. An algebraic equation is an equality involving variables. It says that the value of the expression on one side of the equality sign is equal to the value of the expression on the other side.

2. The equations we study in Classes VI, VII and VIII are linear equations in one variable. In such equations, the expressions which form the equation contain only one variable. Further, the equations are linear, i.e., the highest power of the variable appearing in the equation is 1.

3. A linear equation may have for its solution any rational number.

4. An equation may have linear expressions on both sides. Equations that we studied in Classes VI and VII had just a number on one side of the equation.

5. Just as numbers, variables can, also, be transposed from one side of the equation to the other.

6. Occasionally, the expressions forming equations have to be simplified before we can solve them by usual methods. Some equations may not even be linear to begin with, but they can be brought to a linear form by multiplying both sides of the equation by a suitable expression.

7. The utility of linear equations is in their diverse applications; different problems on numbers, ages, perimeters, combination of currency notes, and so on can be solved using linear equations.

Understanding Quadrilaterals

0852CH03

3.1 Introduction

You know that the paper is a model for a **plane surface**. When you join a number of points without lifting a pencil from the paper (and without retracing any portion of the drawing other than single points), you get a **plane curve**.

Try to recall different varieties of curves you have seen in the earlier classes.

Match the following: (Caution! A figure may match to more than one type).

Figure		Type
(1)		(a) Simple closed curve
(2)		(b) A closed curve that is not simple
(3)		(c) Simple curve that is not closed
(4)		(d) Not a simple curve

Compare your matchings with those of your friends. Do they agree?

3.2 Polygons

A simple closed curve made up of only line segments is called a **polygon**.

Curves that are polygons   Curves that are not polygons
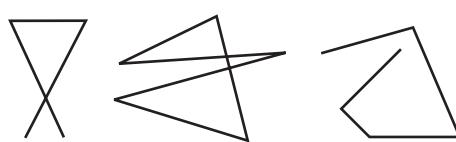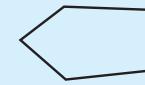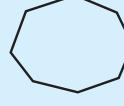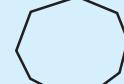
Try to give a few more examples and non-examples for a polygon.

Draw a rough figure of a polygon and identify its sides and vertices.

3.2.1 Classification of polygons

We classify polygons according to the number of sides (or vertices) they have.

Number of sides or vertices	Classification	Sample figure
3	Triangle	
4	Quadrilateral	
5	Pentagon	
6	Hexagon	
7	Heptagon	
8	Octagon	
9	Nonagon	
10	Decagon	
⋮	⋮	⋮
n	n-gon	

3.2.2 Diagonals

A diagonal is a line segment connecting two non-consecutive vertices of a polygon (Fig 3.1).
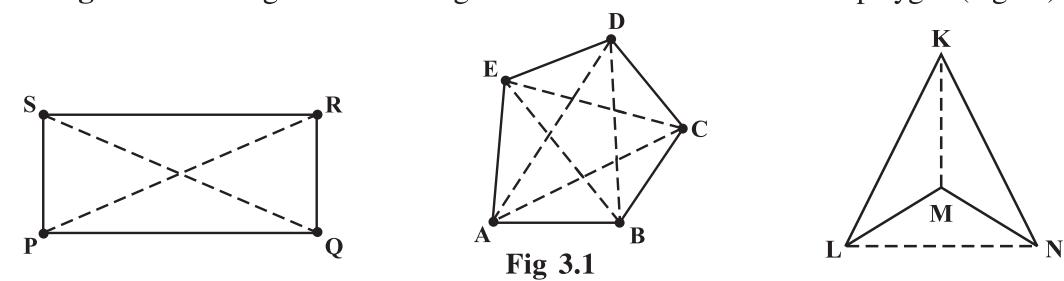
Fig 3.1

Can you name the diagonals in each of the above figures? (Fig 3.1)

Is $\overline{PQ}$ a diagonal? What about $\overline{LN}$?

You already know what we mean by **interior** and **exterior** of a closed curve (Fig 3.2).
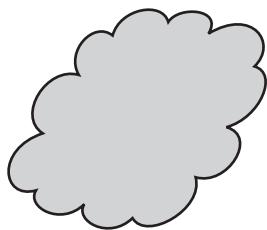
Interior   **Fig 3.2**   **Exterior**
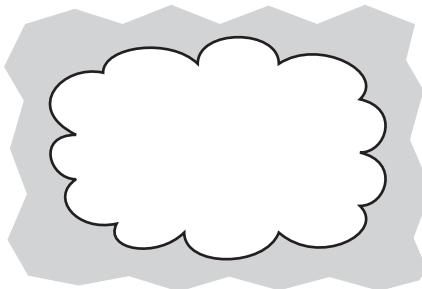
The interior has a boundary. Does the exterior have a boundary? Discuss with your friends.

3.2.3 Convex and concave polygons

Here are some convex polygons and some concave polygons. (Fig 3.3)
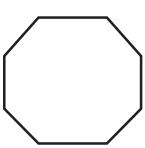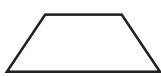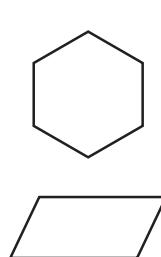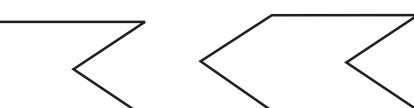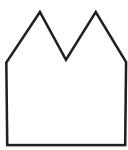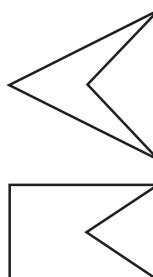
Convex polygons   **Fig 3.3**   **Concave polygons**

   Can you find how these types of polygons differ from one another? Polygons that are convex have no portions of their diagonals in their exteriors or any line segment joining any two different points, in the interior of the polygon, lies wholly in the interior of it . Is this true with concave polygons? Study the figures given. Then try to describe in your own words what we mean by a convex polygon and what we mean by a concave polygon. Give two rough sketches of each kind.

In our work in this class, we will be dealing with convex polygons only.

3.2.4 Regular and irregular polygons

A regular polygon is both 'equiangular' and 'equilateral'. For example, a square has sides of equal length and angles of equal measure. Hence it is a regular polygon. A rectangle is equiangular but not equilateral. Is a rectangle a regular polygon? Is an equilateral triangle a regular polygon? Why?
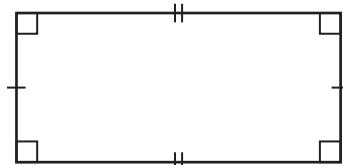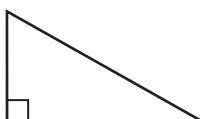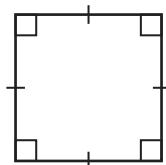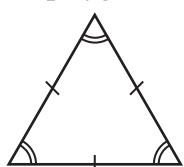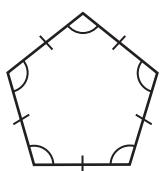
Regular polygons
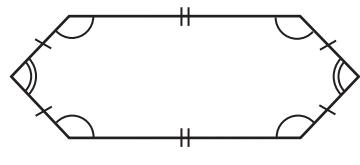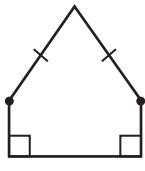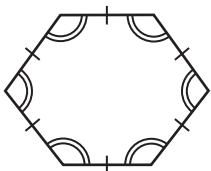
Polygons that are not regular

[**Note:** Use of ⟨⟩ or ⟨⟩ indicates segments of equal length].

In the previous classes, have you come across any quadrilateral that is equilateral but not equiangular? Recall the quadrilateral shapes you saw in earlier classes – Rectangle, Square, Rhombus etc.

Is there a triangle that is equilateral but not equiangular?

3.2.5 Angle sum property

Do you remember the angle-sum property of a triangle? The sum of the measures of the three angles of a triangle is 180°. Recall the methods by which we tried to visualise this fact. We now extend these ideas to a quadrilateral.

DO THIS

1. Take any quadrilateral, say ABCD (Fig 3.4). Divide it into two triangles, by drawing a diagonal. You get six angles 1, 2, 3, 4, 5 and 6.

    Use the angle-sum property of a triangle and argue how the sum of the measures of ∠A, ∠B, ∠C and ∠D amounts to 180° + 180° = 360°.

Fig 3.4

2. Take four congruent card-board copies of any quadrilateral ABCD, with angles as shown [Fig 3.5 (i)]. Arrange the copies as shown in the figure, where angles ∠1, ∠2, ∠3, ∠4 meet at a point [Fig 3.5 (ii)].
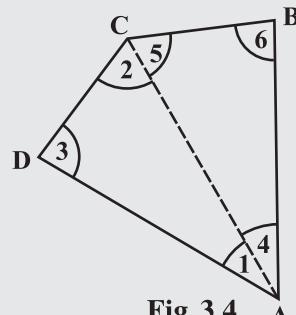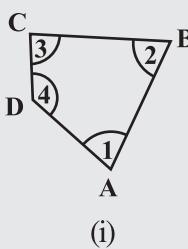
(i)
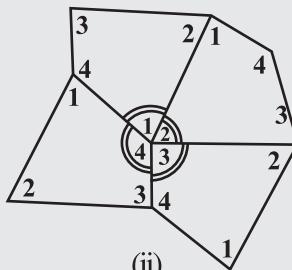
Fig 3.5

(ii)

For doing this you may have to turn and match appropriate corners so that they fit.

What can you say about the sum of the angles ∠1, ∠2, ∠3 and ∠4?

[**Note:** We denote the angles by ∠1, ∠2, ∠3, etc., and their respective measures by $m\angle 1$, $m\angle 2$, $m\angle 3$, etc.]

The sum of the measures of the four angles of a quadrilateral is_____.

You may arrive at this result in several other ways also.

3. As before consider quadrilateral ABCD (Fig 3.6). Let P be any point in its interior. Join P to vertices A, B, C and D. In the figure, consider ΔPAB. From this we see $x = 180° - m\angle 2 - m\angle 3$; similarly from ΔPBC, $y = 180° - m\angle 4 - m\angle 5$, from ΔPCD, $z = 180° - m\angle 6 - m\angle 7$ and from ΔPDA, $w = 180° - m\angle 8 - m\angle 1$. Use this to find the total measure $m\angle 1 + m\angle 2 + ... + m\angle 8$, does it help you to arrive at the result? Remember $\angle x + \angle y + \angle z + \angle w = 360°$.

4. These quadrilaterals were convex. What would happen if the quadrilateral is not convex? Consider quadrilateral ABCD. Split it into two triangles and find the sum of the interior angles (Fig 3.7).
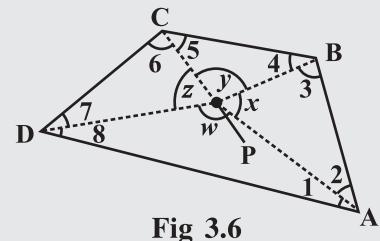
Fig 3.6
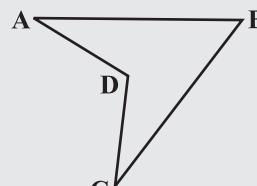
Fig 3.7

EXERCISE 3.1

1. Given here are some figures.
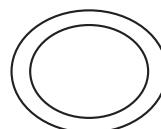
(1)   (2)   (3)   (4)
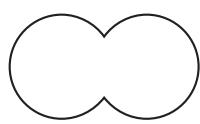
(5)   (6)   (7)   (8)

Classify each of them on the basis of the following.
   (a) Simple curve   (b) Simple closed curve   (c) Polygon
   (d) Convex polygon   (e) Concave polygon

2. How many diagonals does each of the following have?
   (a) A convex quadrilateral   (b) A regular hexagon   (c) A triangle

3. What is the sum of the measures of the angles of a convex quadrilateral? Will this property hold if the quadrilateral is not convex? (Make a non-convex quadrilateral and try!)

4. Examine the table. (Each figure is divided into triangles and the sum of the angles deduced from that.)

Figure				
Side	3	4	5	6
Angle sum	180°	$2 \times 180°$ $= (4-2) \times 180°$	$3 \times 180°$ $= (5-2) \times 180°$	$4 \times 180°$ $= (6-2) \times 180°$

What can you say about the angle sum of a convex polygon with number of sides?

(a) 7   (b) 8   (c) 10   (d) n

5. What is a regular polygon?

State the name of a regular polygon of

(i) 3 sides   (ii) 4 sides   (iii) 6 sides

6. Find the angle measure x in the following figures.
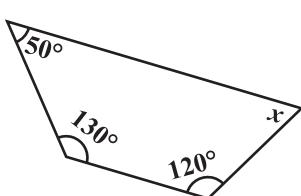
(a)
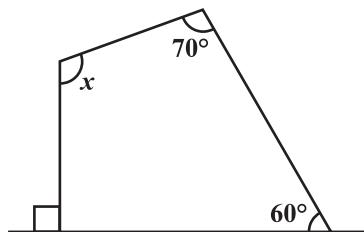
(b)
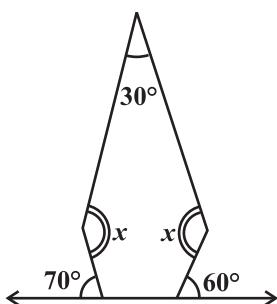
(c)
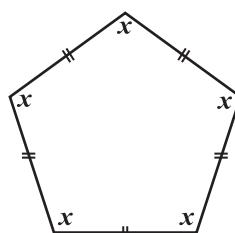
(d)

7.
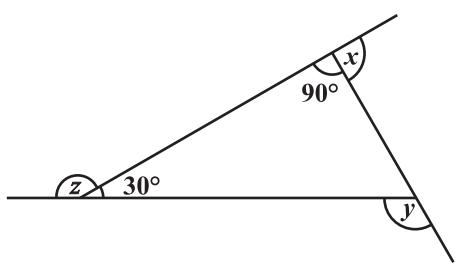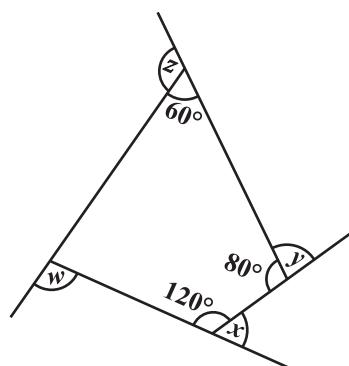
(a) Find $x + y + z$ (b) Find $x + y + z + w$

3.3 Sum of the Measures of the Exterior Angles of a Polygon

On many occasions a knowledge of exterior angles may throw light on the nature of interior angles and sides.

DO THIS

Draw a polygon on the floor, using a piece of chalk. (In the figure, a pentagon ABCDE is shown) (Fig 3.8).

We want to know the total measure of angles, i.e, $m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 + m\angle 5$. Start at A. Walk along $\overline{AB}$. On reaching B, you need to turn through an angle of $m\angle 1$, to walk along $\overline{BC}$. When you reach at C, you need to turn through an angle of $m\angle 2$ to walk along $\overline{CD}$. You continue to move in this manner, until you return to side AB. You would have in fact made one complete turn.

Therefore, $m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 + m\angle 5 = 360°$

This is true whatever be the number of sides of the polygon.

Therefore, *the sum of the measures of the external angles of any polygon is 360°.*
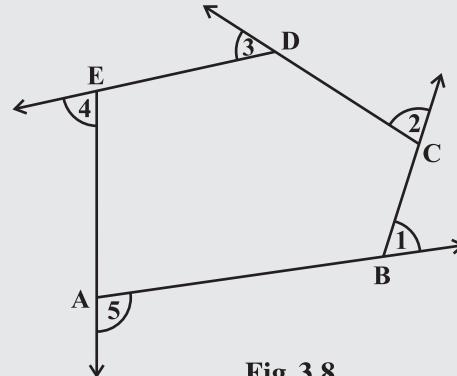
Fig 3.8

Example 1: Find measure x in Fig 3.9.

Solution:
$$x + 90° + 50° + 110° = 360° \quad \text{(Why?)}$$
$$x + 250° = 360°$$
$$x = 110°$$
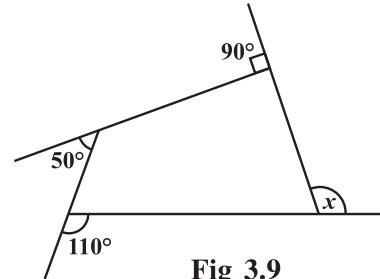
Fig 3.9

TRY THESE

Take a regular hexagon Fig 3.10.

1. What is the sum of the measures of its exterior angles x, y, z, p, q, r?
2. Is $x = y = z = p = q = r$? Why?
3. What is the measure of each?
    (i) exterior angle
    (ii) interior angle

4. Repeat this activity for the cases of
    (i) a regular octagon
    (ii) a regular 20-gon
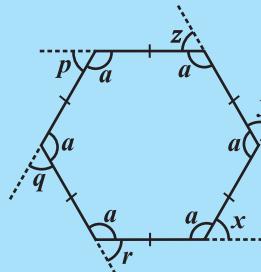
Fig 3.10

Example 2: Find the number of sides of a regular polygon whose each exterior angle has a measure of 45°.

Solution: Total measure of all exterior angles = 360°

Measure of each exterior angle = 45°

Therefore, the number of exterior angles = $\dfrac{360}{45} = 8$

The polygon has 8 sides.

EXERCISE 3.2

1. Find x in the following figures.

(a)   (b)

2. Find the measure of each exterior angle of a regular polygon of
   (i) 9 sides   (ii) 15 sides
3. How many sides does a regular polygon have if the measure of an exterior angle is 24°?
4. How many sides does a regular polygon have if each of its interior angles is 165°?
5. (a) Is it possible to have a regular polygon with measure of each exterior angle as 22°?
   (b) Can it be an interior angle of a regular polygon? Why?
6. (a) What is the minimum interior angle possible for a regular polygon? Why?
   (b) What is the maximum exterior angle possible for a regular polygon?

3.4 Kinds of Quadrilaterals

Based on the nature of the sides or angles of a quadrilateral, it gets special names.

3.4.1 Trapezium

Trapezium is a quadrilateral with a pair of parallel sides.
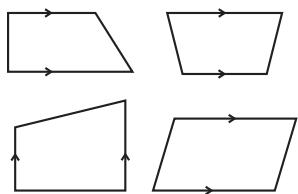
These are trapeziums   **These are not trapeziums**
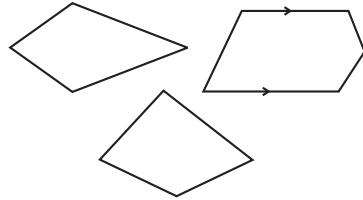
Study the above figures and discuss with your friends why some of them are trapeziums while some are not. (**Note:** *The arrow marks indicate parallel lines*).

DO THIS

1. Take identical cut-outs of congruent triangles of sides 3 cm, 4 cm, 5 cm. Arrange them as shown (Fig 3.11).
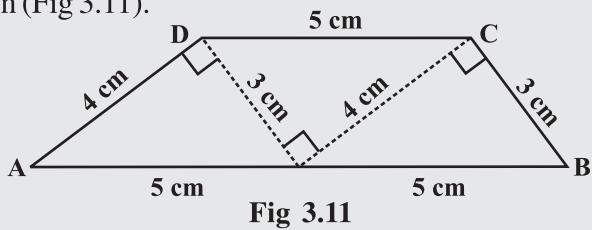
Fig 3.11

You get a trapezium. (Check it!) Which are the parallel sides here? Should the non-parallel sides be equal?

You can get two more trapeziums using the same set of triangles. Find them out and discuss their shapes.

2. Take four set-squares from your and your friend's instrument boxes. Use different numbers of them to place side by-side and obtain different trapeziums.

If the non-parallel sides of a trapezium are of equal length, we call it an *isosceles trapezium*. Did you get an isoceles trapezium in any of your investigations given above?

3.4.2 Kite

Kite is a special type of a quadrilateral. The sides with the same markings in each figure are equal. For example AB = AD and BC = CD.
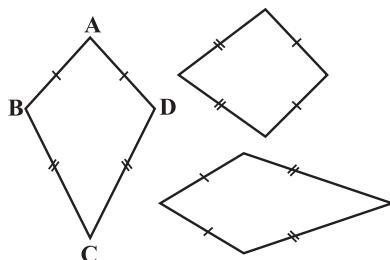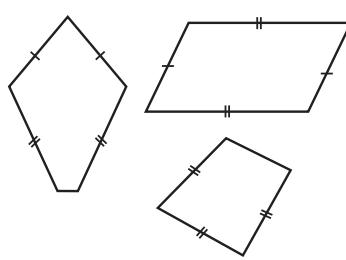
| These are kites | These are not kites |

Study these figures and try to describe what a kite is. Observe that

(i)   A kite has 4 sides (It is a quadrilateral).

(ii)   There are exactly two **distinct consecutive pairs** of sides of equal length.

Check whether a square is a kite.

DO THIS

Take a thick white sheet.

Fold the paper once.

Draw two line segments of different lengths as shown in Fig 3.12.

Cut along the line segments and open up.

You have the shape of a kite (Fig 3.13).

Has the kite any line symmetry?

Fold both the diagonals of the kite. Use the set-square to check if they cut at right angles. Are the diagonals equal in length?

Verify (by paper-folding or measurement) if the diagonals bisect each other.

By folding an angle of the kite on its opposite, check for angles of equal measure.

Observe the diagonal folds; do they indicate any diagonal being an angle bisector?

Share your findings with others and list them. A summary of these results are given elsewhere in the chapter for your reference.
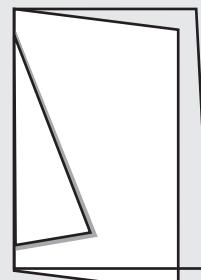
Fig 3.12

Show that ΔABC and ΔADC are congruent . What do we infer from this?
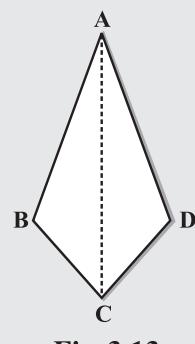
Fig 3.13

3.4.3 Parallelogram

A parallelogram is a quadrilateral. As the name suggests, it has something to do with parallel lines.

$\overline{AB} \parallel \overline{DC}$

$\overline{AD} \parallel \overline{BC}$

$\overline{AB} \parallel \overline{CD}$

$\overline{QP} \parallel \overline{SR}$

$\overline{QS} \parallel \overline{PR}$

$\overline{LM} \parallel \overline{ON}$

$\overline{LO} \parallel \overline{MN}$

$\overline{AB} \parallel \overline{ED}$

$\overline{BC} \parallel \overline{FE}$
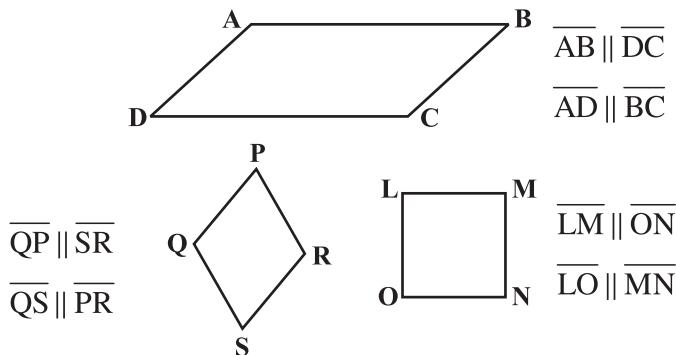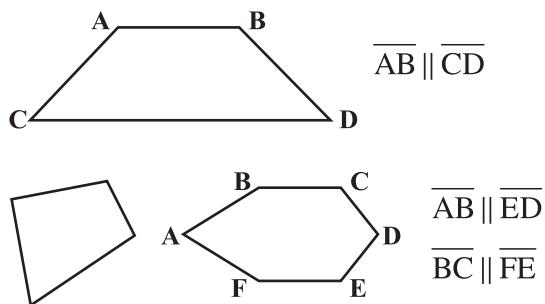
These are parallelograms   **These are not parallelograms**

Study these figures and try to describe in your own words what we mean by a parallelogram. Share your observations with your friends.

Check whether a rectangle is also a parallelogram.

DO THIS

Take two different rectangular cardboard strips of different widths (Fig 3.14).

Strip 1   **Fig 3.14**   **Strip 2**
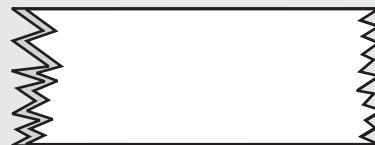
Place one strip horizontally and draw lines along its edge as drawn in the figure (Fig 3.15).

Now place the other strip in a slant position over the lines drawn and use this to draw two more lines as shown (Fig 3.16).

Fig 3.15

These four lines enclose a quadrilateral. This is made up of two pairs of parallel lines (Fig 3.17).
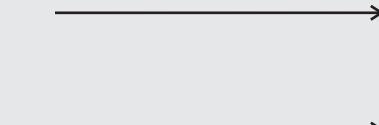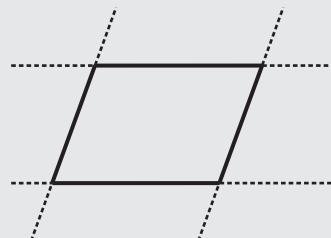
Fig 3.16   **Fig 3.17**

It is a parallelogram.

A parallelogram is a quadrilateral whose opposite sides are parallel.

3.4.4 Elements of a parallelogram

There are four sides and four angles in a parallelogram. Some of these are equal. There are some terms associated with these elements that you need to remember.

Given a parallelogram ABCD (Fig 3.18).
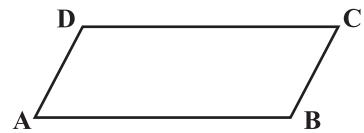
Fig 3.18

$\overline{AB}$ and $\overline{DC}$, are **opposite sides**. $\overline{AD}$ and $\overline{BC}$ form another pair of opposite sides.

∠A and ∠C are a pair of **opposite angles**; another pair of opposite angles would be ∠B and ∠D.

$\overline{AB}$ and $\overline{BC}$ are **adjacent sides**. This means, one of the sides starts where the other ends. Are $\overline{BC}$ and $\overline{CD}$ adjacent sides too? Try to find two more pairs of adjacent sides.

∠A and ∠B are **adjacent angles**. They are at the ends of the same side. ∠B and ∠C are also adjacent. Identify other pairs of adjacent angles of the parallelogram.

DO THIS

Take cut-outs of two identical parallelograms, say ABCD and A′B′C′D′ (Fig 3.19).
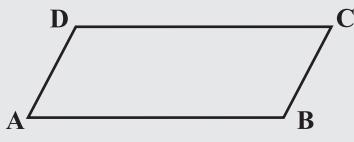
Fig 3.19
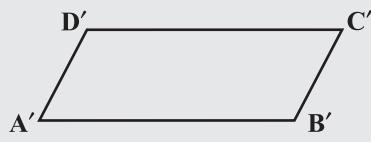
Here $\overline{AB}$ is same as $\overline{A'B'}$ except for the name. Similarly the other corresponding sides are equal too.

Place $\overline{A'B'}$ over $\overline{DC}$. Do they coincide? What can you now say about the lengths $\overline{AB}$ and $\overline{DC}$?

Similarly examine the lengths $\overline{AD}$ and $\overline{BC}$. What do you find?

You may also arrive at this result by measuring $\overline{AB}$ and $\overline{DC}$.

Property: *The opposite sides of a parallelogram are of equal length.*

TRY THESE

Take two identical set squares with angles 30° – 60° – 90° and place them adjacently to form a parallelogram as shown in Fig 3.20. Does this help you to verify the above property?

You can further strengthen this idea through a logical argument also.

Consider a parallelogram ABCD (Fig 3.21). Draw any one diagonal, say $\overline{AC}$.
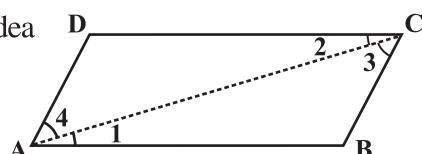
Fig 3.21
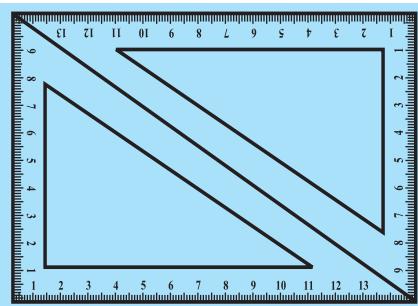
Fig 3.20

Looking at the angles,

$$\angle 1 = \angle 2 \quad \text{and} \quad \angle 3 = \angle 4 \quad \text{(Why?)}$$

Since in triangles ABC and ADC, $\angle 1 = \angle 2, \ \angle 3 = \angle 4$

and $\overline{AC}$ is common, so, by ASA congruency condition,

$$\Delta \ ABC \quad \cong \quad \Delta \ CDA \qquad \text{(How is ASA used here?)}$$

This gives $\qquad\qquad AB = DC \quad \text{and} \quad BC = AD$.

Example 3: Find the perimeter of the parallelogram PQRS (Fig 3.22).

Solution: In a parallelogram, the opposite sides have same length.

Therefore, PQ = SR = 12 cm   and   QR = PS = 7 cm

So, Perimeter = PQ + QR + RS + SP

$\qquad\qquad$ = 12 cm + 7 cm + 12 cm + 7 cm = 38 cm
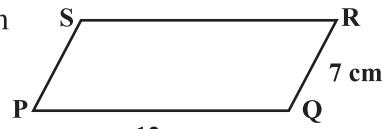
Fig 3.22

3.4.5 Angles of a parallelogram

We studied a property of parallelograms concerning the (opposite) sides. What can we say about the angles?

DO THIS

Let ABCD be a parallelogram (Fig 3.23). Copy it on a tracing sheet. Name this copy as A′B′C′D′. Place A′B′C′D′ on ABCD. Pin them together at the point where the diagonals meet. Rotate the transparent sheet by 180°. The parallelograms still concide; but you now find A′ lying exactly on C and vice-versa; similarly B′ lies on D and vice-versa.
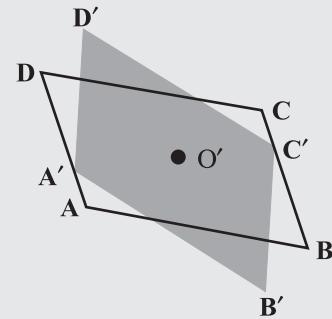
Fig 3.23

Does this tell you anything about the measures of the angles A and C? Examine the same for angles B and D. State your findings.

Property: *The opposite angles of a parallelogram are of equal measure.*

TRY THESE

Take two identical 30° – 60° – 90° set-squares and form a parallelogram as before. Does the figure obtained help you to confirm the above property?

You can further justify this idea through logical arguments.

If $\overline{AC}$ and $\overline{BD}$ are the diagonals of the parallelogram, (Fig 3.24) you find that

$$\angle 1 = \angle 2 \quad \text{and} \quad \angle 3 = \angle 4 \quad \text{(Why?)}$$
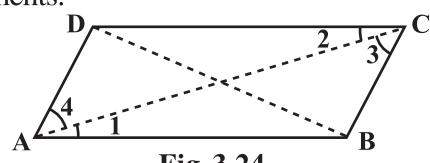
Fig 3.24

Studying $\triangle$ ABC and $\triangle$ ADC (Fig 3.25) separately, will help you to see that by ASA congruency condition,

$$\triangle \text{ ABC} \quad \cong \quad \triangle \text{ CDA} \qquad \text{(How?)}$$
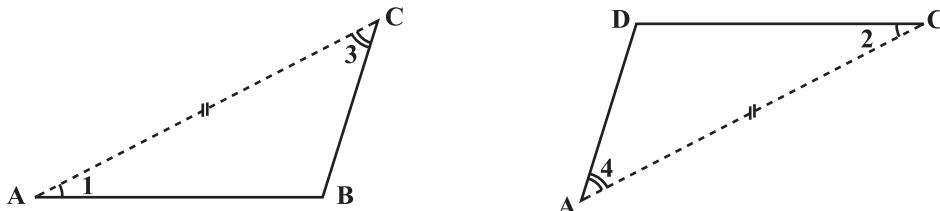
Fig 3.25

This shows that ∠B and ∠D have same measure. In the same way you can get $m\angle A = m\angle C$.

Alternatively, $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$, we have, $m\angle A = \angle 1 + \angle 4 = \angle 2 + \angle C \, m\angle C$

Example 4: In Fig 3.26, BEST is a parallelogram. Find the values x, y and z.

Solution: S is opposite to B.

So, $\qquad x = 100°$ (opposite angles property)

$\qquad y = 100°$ (measure of angle corresponding to ∠x)

$\qquad z = 80°$ (since ∠y, ∠z is a linear pair)

Fig 3.26

We now turn our attention to adjacent angles of a parallelogram. In parallelogram ABCD, (Fig 3.27).

∠A and ∠D are supplementary since $\overline{DC} \parallel \overline{AB}$ and with transversal $\overline{DA}$, these two angles are interior opposite.

∠A and ∠B are also supplementary. Can you say 'why'?

Fig 3.27

$\overline{AD} \parallel \overline{BC}$ and $\overline{BA}$ is a transversal, making ∠A and ∠B interior opposite.

Identify two more pairs of supplementary angles from the figure.

Property: *The adjacent angles in a parallelogram are supplementary.*

Example 5: In a parallelogram RING, (Fig 3.28) if $m\angle R = 70°$, find all the other angles.

Solution: Given $\quad m\angle R = 70°$

Then $\qquad\qquad\qquad m\angle N = 70°$

because ∠R and ∠N are opposite angles of a parallelogram.

Since ∠R and ∠I are supplementary,

Fig 3.28

$$m\angle I = 180° - 70° = 110°$$

Also, $\qquad\qquad\qquad m\angle G = 110°$ since ∠G is opposite to ∠I

Thus, $\qquad\qquad\qquad m\angle R = m\angle N = 70°$ and $m\angle I = m\angle G = 110°$
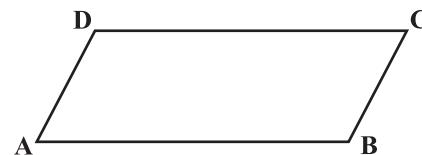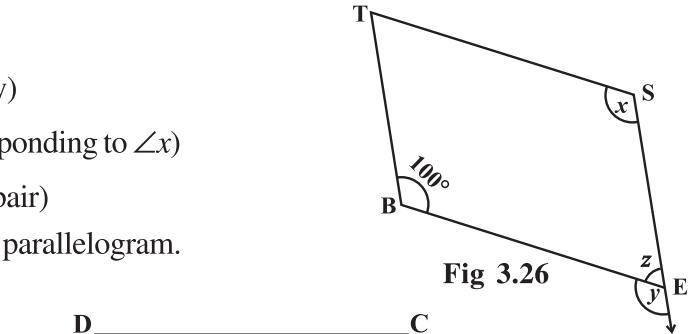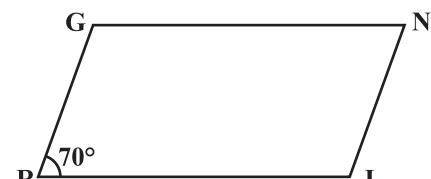
After showing $m\angle R = m\angle N = 70°$, can you find $m\angle I$ and $m\angle G$ by any other method?

3.4.6 Diagonals of a parallelogram

The diagonals of a parallelogram, in general, are not of equal length. (Did you check this in your earlier activity?) However, the diagonals of a parallelogram have an interesting property.

DO THIS

Take a cut-out of a parallelogram, say,

ABCD (Fig 3.29). Let its diagonals $\overline{AC}$ and $\overline{DB}$ meet at O.
      Find the mid point of $\overline{AC}$ by a fold, placing C on A. Is the mid-point same as O?
      Does this show that diagonal $\overline{DB}$ bisects the diagonal $\overline{AC}$ at the point O? Discuss it with your friends. Repeat the activity to find where the mid point of $\overline{DB}$ could lie.

Fig 3.29
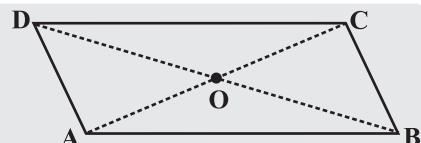
Property: *The diagonals of a parallelogram bisect each other (at the point of their intersection, of course!)*

To argue and justify this property is not very difficult. From Fig 3.30, applying ASA criterion, it is easy to see that

$$\Delta AOB \cong \Delta COD \quad \text{(How is ASA used here?)}$$

Fig 3.30

This gives   AO = CO   and   BO = DO

Example 6: In Fig 3.31 HELP is a parallelogram. (Lengths are in cms). Given that OE = 4 and HL is 5 more than PE? Find OH.

Solution : If   OE = 4 then OP also is 4 (Why?)

So   PE = 8,   (Why?)

Therefore   HL = 8 + 5 = 13

Fig 3.31

Hence   OH = $\dfrac{1}{2} \times 13 = 6.5$ (cms)
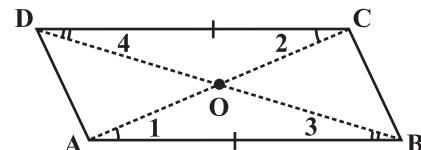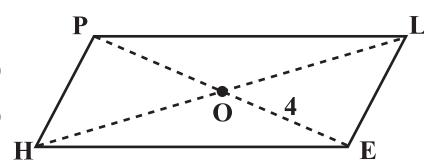
EXERCISE 3.3

1. Given a parallelogram ABCD. Complete each statement along with the definition or property used.

    (i)   AD = ...   (ii) $\angle$ DCB = ...

    (iii)   OC = ...   (iv) $m \angle DAB + m \angle CDA = ...$
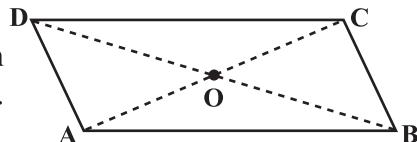
2. Consider the following parallelograms. Find the values of the unknowns x, y, z.
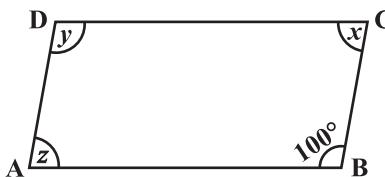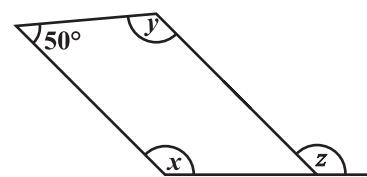
(i)   (ii)
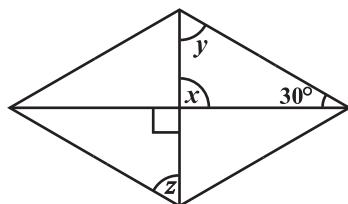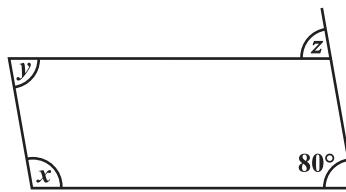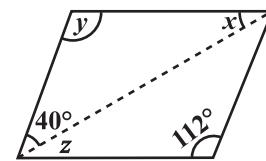
(iii)   (iv)   (v)

3. Can a quadrilateral ABCD be a parallelogram if
  (i)   ∠D + ∠B = 180°?   (ii)   AB = DC = 8 cm, AD = 4 cm and BC = 4.4 cm?
  (iii)   ∠A = 70° and ∠C = 65°?

4. Draw a rough figure of a quadrilateral that is not a parallelogram but has exactly two opposite angles of equal measure.

5. The measures of two adjacent angles of a parallelogram are in the ratio 3 : 2. Find the measure of each of the angles of the parallelogram.

6. Two adjacent angles of a parallelogram have equal measure. Find the measure of each of the angles of the parallelogram.

7. The adjacent figure HOPE is a parallelogram. Find the angle measures x, y and z. State the properties you use to find them.

8. The following figures GUNS and RUNS are parallelograms. Find x and y. (Lengths are in cm)
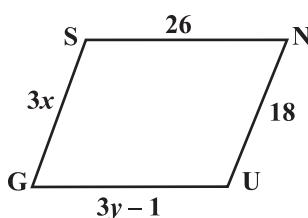
(i)   (ii)
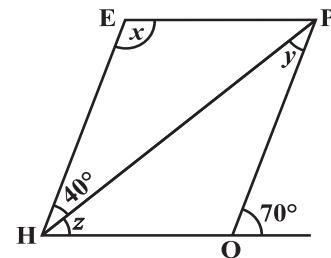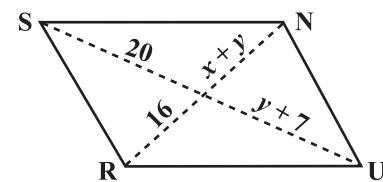
9.
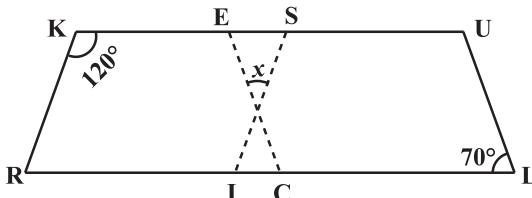
In the above figure both RISK and CLUE are parallelograms. Find the value of x.

10. Explain how this figure is a trapezium. Which of its two sides are parallel? (Fig 3.32)
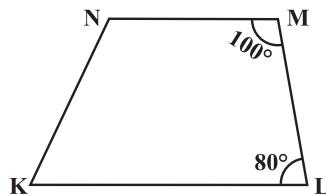
Fig 3.32
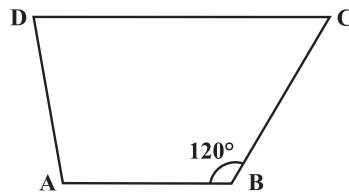
Fig 3.33

Fig 3.34

11. Find $m\angle C$ in Fig 3.33 if $\overline{AB} \parallel \overline{DC}$.

12. Find the measure of $\angle P$ and $\angle S$ if $\overline{SP} \parallel \overline{RQ}$ in Fig 3.34. (If you find $m\angle R$, is there more than one method to find $m\angle P$?)
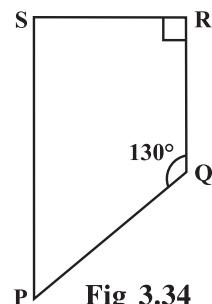
3.5 Some Special Parallelograms

3.5.1 Rhombus

We obtain a Rhombus (which, you will see, is a parallelogram) as a special case of kite (which is not a a parallelogram).

DO THIS

Recall the paper-cut kite you made earlier.
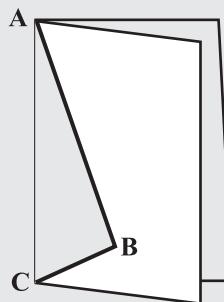
Kite-cut
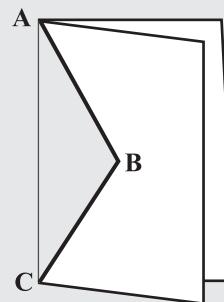
Rhombus-cut

When you cut along ABC and opened up, you got a kite. Here lengths AB and BC were different. If you draw AB = BC, then the kite you obtain is called a **rhombus**.

Note that the sides of rhombus are all of same length; this is not the case with the kite.

A rhombus is a quadrilateral with sides of equal length.

Since the opposite sides of a rhombus have the same length, it is also a parallelogram. So, *a rhombus has all the properties of a parallelogram and also that of a kite*. Try to list them out. You can then verify your list with the check list summarised in the book elsewhere.
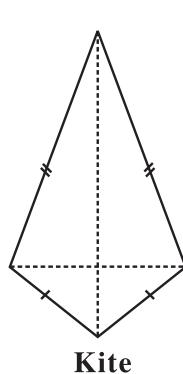
Kite
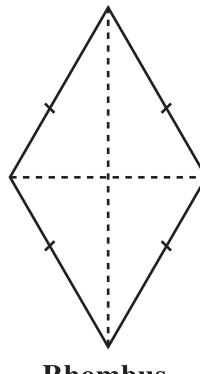
Rhombus

The most useful property of a rhombus is that of its diagonals.

Property: *The diagonals of a rhombus are perpendicular bisectors of one another.*

Take a copy of rhombus. By paper-folding verify if the point of intersection is the mid-point of each diagonal. You may also check if they intersect at right angles, using the corner of a set-square.

Here is an outline justifying this property using logical steps.

ABCD is a rhombus (Fig 3.35). Therefore it is a parallelogram too.

Since diagonals bisect each other,   OA = OC and OB = OD.

We have to show that $m\angle AOD = m\angle COD = 90°$

It can be seen that by SSS congruency criterion

$$\Delta AOD \quad \cong \quad \Delta COD$$

Fig 3.35

Therefore, $m\angle AOD = m\angle COD$

Since $\angle AOD$ and $\angle COD$ are a linear pair,

$$m\angle AOD = m\angle COD = 90°$$
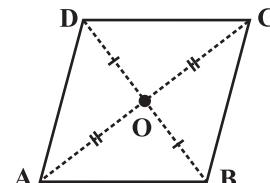
| Since AO = CO   (Why?) |
| AD = CD   (Why?) |
| OD = OD |
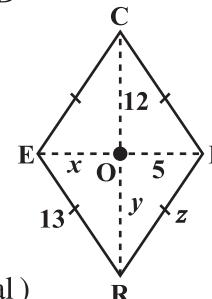
Example 7:

RICE is a rhombus (Fig 3.36). Find x, y, z. Justify your findings.

Solution:

$x =$ OE	$y =$ OR	$z =$ side of the rhombus
= OI (diagonals bisect)	= OC (diagonals bisect)	= 13 (all sides are equal)
= 5	= 12	

Fig 3.36

3.5.2 A rectangle

A rectangle is a parallelogram with equal angles (Fig 3.37).

What is the full meaning of this definition? Discuss with your friends.

If the rectangle is to be equiangular, what could be the measure of each angle?

Fig 3.37

Let the measure of each angle be $x°$.

Then $4x° = 360°$ (Why)?

Therefore, $x° = 90°$

Thus each angle of a rectangle is a right angle.

So, a rectangle is a parallelogram in which every angle is a right angle.

     Being a parallelogram, the rectangle has opposite sides of equal length and its diagonals bisect each other.

In a parallelogram, the diagonals can be of different lengths. (Check this); but surprisingly the rectangle (being a special case) has diagonals of equal length.

Property: *The diagonals of a rectangle are of equal length.*
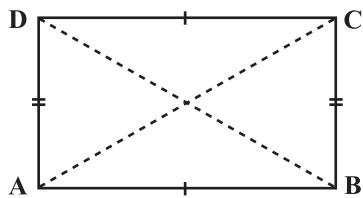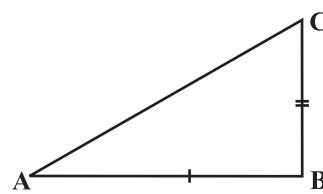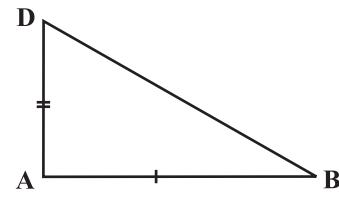
Fig 3.38   Fig 3.39   Fig 3.40

This is easy to justify. If ABCD is a rectangle (Fig 3.38), then looking at triangles ABC and ABD separately [(Fig 3.39) and (Fig 3.40) respectively], we have

$$\Delta \text{ABC} \cong \Delta \text{ABD}$$

This is because

AB = AB	(Common)
BC = AD	(Why?)
$m \angle A = m \angle B = 90°$	(Why?)

The congruency follows by SAS criterion.

Thus   AC = BD

and *in a rectangle the diagonals, besides being equal in length bisect each other* (Why?)

Example 8: RENT is a rectangle (Fig 3.41). Its diagonals meet at O. Find x, if OR = $2x + 4$ and OT = $3x + 1$.

Solution: $\overline{\text{OT}}$ is half of the diagonal $\overline{\text{TE}}$,

$\overline{\text{OR}}$ is half of the diagonal $\overline{\text{RN}}$.

Diagonals are equal here. (Why?)

So, their halves are also equal.

Therefore $3x + 1 = 2x + 4$
or $x = 3$

3.5.3 A square

A square is a rectangle with equal sides.

This means a square has all the properties of a rectangle with an additional requirement that all the sides have equal length.

The square, like the rectangle, has diagonals of equal length.

In a rectangle, there is no requirement for the diagonals to be perpendicular to one another, (Check this).
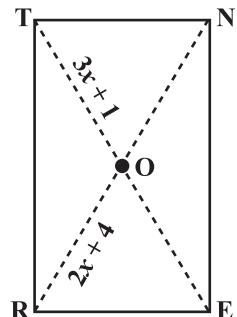
Fig 3.41
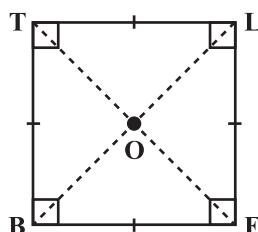
BELT is a square, BE = EL = LT = TB
∠B, ∠E, ∠L, ∠T are right angles.

BL = ET and $\overline{\text{BL}} \perp \overline{\text{ET}}$.
  OB = OL and OE = OT.

In a square the diagonals.

  (i)   bisect one another   (square being a parallelogram)
 (ii)   are of equal length   (square being a rectangle) and
(iii)   are perpendicular to one another.

Hence, we get the following property.

Property: *The diagonals of a square are perpendicular bisectors of each other.*

DO THIS

Take a square sheet, say PQRS (Fig 3.42).

Fold along both the diagonals. Are their mid-points the same?

Check if the angle at O is 90° by using a set-square.

This verifies the property stated above.
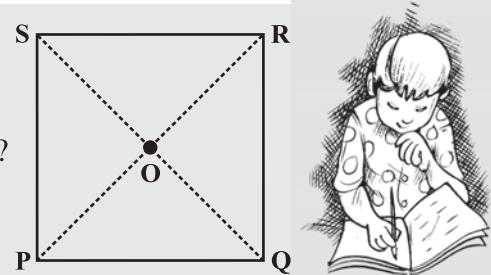
Fig 3.42

We can justify this also by arguing logically:

ABCD is a square whose diagonals meet at O (Fig 3.43).

$$OA = OC \quad \text{(Since the square is a parallelogram)}$$

By SSS congruency condition, we now see that

$$\Delta\,AOD \cong \Delta\,COD \quad \text{(How?)}$$

Therefore, $m\angle AOD = m\angle COD$

These angles being a linear pair, each is right angle.
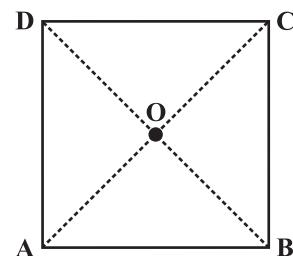
Fig 3.43

EXERCISE 3.4

1. State whether True or False.

    (a)   All rectangles are squares
    (b)   All rhombuses are parallelograms
    (c)   All squares are rhombuses and also rectangles
    (d)   All squares are not parallelograms.
    (e)   All kites are rhombuses.
    (f)   All rhombuses are kites.
    (g)   All parallelograms are trapeziums.
    (h)   All squares are trapeziums.

2. Identify all the quadrilaterals that have.

    (a)   four sides of equal length   (b)   four right angles

3. Explain how a square is.

     (i)   a quadrilateral   (ii)   a parallelogram   (iii)   a rhombus   (iv)   a rectangle

4. Name the quadrilaterals whose diagonals.

     (i)   bisect each other   (ii)   are perpendicular bisectors of each other   (iii)   are equal

5. Explain why a rectangle is a convex quadrilateral.

6. ABC is a right-angled triangle and O is the mid point of the side opposite to the right angle. Explain why O is equidistant from A, B and C. (The dotted lines are drawn additionally to help you).
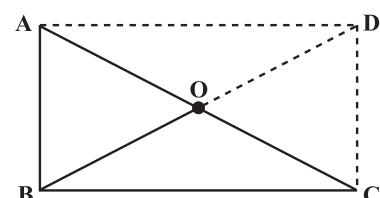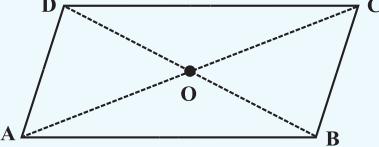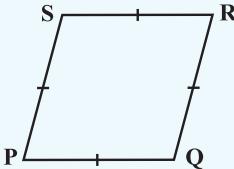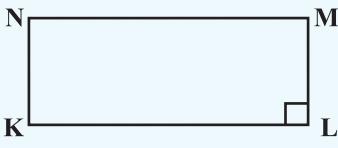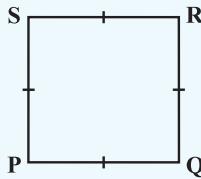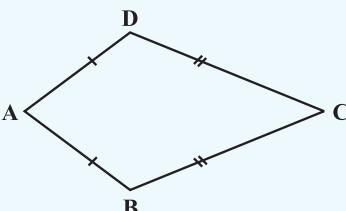
THINK, DISCUSS AND WRITE

1. A mason has made a concrete slab. He needs it to be rectangular. In what different ways can he make sure that it is rectangular?

2. A square was defined as a rectangle with all sides equal. Can we define it as rhombus with equal angles? Explore this idea.

3. Can a trapezium have all angles equal? Can it have all sides equal? Explain.

WHAT HAVE WE DISCUSSED?

Quadrilateral	Properties
Parallelogram: A quadrilateral with each pair of opposite sides parallel.	(1) Opposite sides are equal. (2) Opposite angles are equal. (3) Diagonals bisect one another.
Rhombus: A parallelogram with sides of equal length.	(1) All the properties of a parallelogram. (2) Diagonals are perpendicular to each other.
Rectangle: A parallelogram with a right angle.	(1) All the properties of a parallelogram. (2) Each of the angles is a right angle. (3) Diagonals are equal.
Square: A rectangle with sides of equal length.	All the properties of a parallelogram, rhombus and a rectangle.
Kite: A quadrilateral with exactly two pairs of equal consecutive sides	(1) The diagonals are perpendicular to one another (2) One of the diagonals bisects the other. (3) In the figure $m\angle B = m\angle D$ but $m\angle A \neq m\angle C$.

Practical Geometry

0852CH04

4.1 Introduction

You have learnt how to draw triangles in Class VII. We require three measurements (of sides and angles) to draw a unique triangle.

Since three measurements were enough to draw a triangle, a natural question arises whether four measurements would be sufficient to draw a unique four sided closed figure, namely, a quadrilateral.

DO THIS

Take a pair of sticks of equal lengths, say 10 cm. Take another pair of sticks of equal lengths, say, 8 cm. Hinge them up suitably to get a rectangle of length 10 cm and breadth 8 cm.

This rectangle has been created with the 4 available measurements.

Now just push along the breadth of the rectangle. Is the new shape obtained, still a rectangle (Fig 4.2)? Observe that the rectangle has now become a parallelogram. Have you altered the lengths of the sticks? No! The measurements of sides remain the same.

Give another push to the newly obtained shape in a different direction; what do you get? You again get a parallelogram, which is altogether different (Fig 4.3), yet the four measurements remain the same.

Fig 4.1

Fig 4.2

Fig 4.3

This shows that 4 measurements of a quadrilateral cannot determine it uniquely. Can 5 measurements determine a quadrilateral uniquely? Let us go back to the activity!
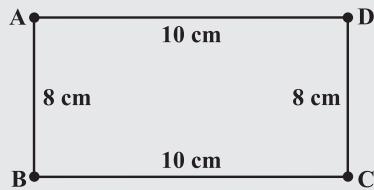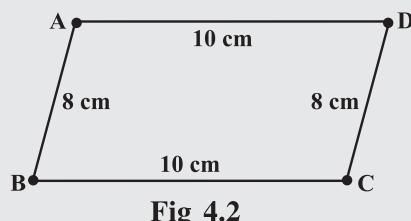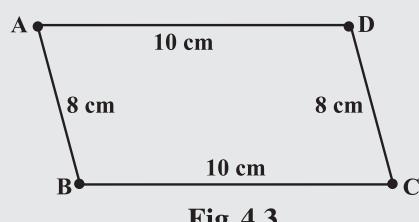
You have constructed a rectangle with two sticks each of length 10 cm and other two sticks each of length 8 cm. Now introduce another stick of length equal to BD and tie it along BD (Fig 4.4). If you push the breadth now, does the shape change? No! It cannot, without making the figure open. The introduction of the fifth stick has fixed the rectangle uniquely, i.e., there is no other quadrilateral (with the given lengths of sides) possible now.

Fig 4.4

Thus, we observe that five measurements can determine a quadrilateral uniquely. But will any five measurements (of sides and angles) be sufficient to draw a unique quadrilateral?
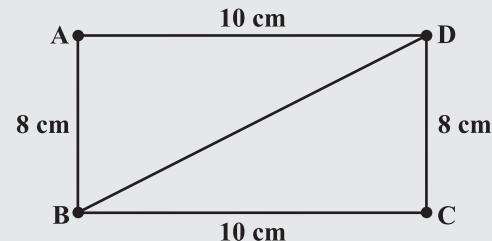
THINK, DISCUSS AND WRITE

Arshad has five measurements of a quadrilateral ABCD. These are AB = 5 cm, ∠A = 50°, AC = 4 cm, BD = 5 cm and AD = 6 cm. Can he construct a unique quadrilateral? Give reasons for your answer.

4.2 Constructing a Quadrilateral

We shall learn how to construct a unique quadrilateral given the following measurements:

- When four sides and one diagonal are given.
- When two diagonals and three sides are given.
- When two adjacent sides and three angles are given.
- When three sides and two included angles are given.
- When other special properties are known.

Let us take up these constructions one-by-one.

4.2.1 When the lengths of four sides and a diagonal are given

We shall explain this construction through an example.

Example 1: Construct a quadrilateral PQRS where PQ = 4 cm, QR = 6 cm, RS = 5 cm, PS = 5.5 cm and PR = 7 cm.

Solution: [A rough sketch will help us in visualising the quadrilateral. We draw this first and mark the measurements.] (Fig 4.5)
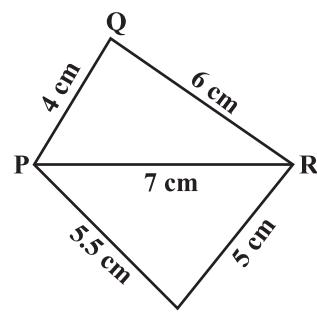
Fig 4.5

Step 1   From the rough sketch, it is easy to see that ΔPQR can be constructed using SSS construction condition. Draw ΔPQR (Fig 4.6).
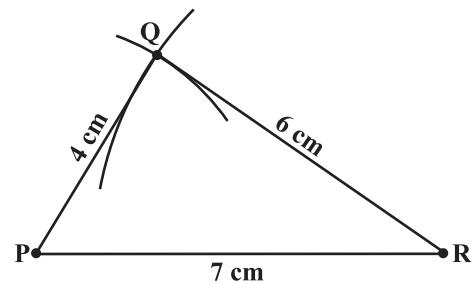
Fig 4.6

Step 2   Now, we have to locate the fourth point S. This 'S' would be on the side opposite to Q with reference to PR. For that, we have two measurements.

S is 5.5 cm away from P. So, with P as centre, draw an arc of radius 5.5 cm. (The point S is somewhere on this arc!) (Fig 4.7).
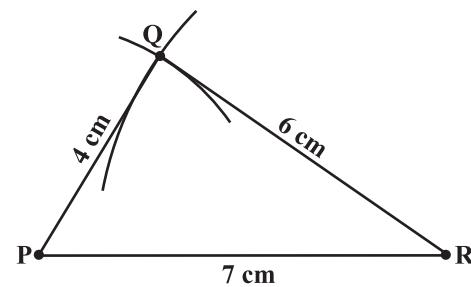
Fig 4.7

Step 3   S is 5 cm away from R. So with R as centre, draw an arc of radius 5 cm (The point S is somewhere on this arc also!) (Fig 4.8).
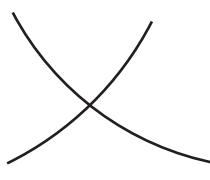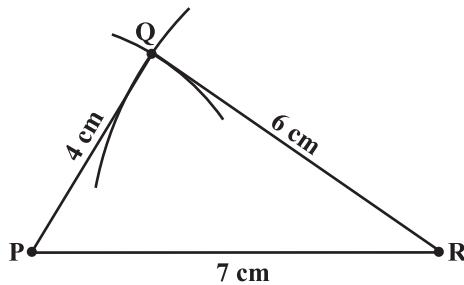
Fig 4.8

Step 4 S should lie on both the arcs drawn. So it is the point of intersection of the two arcs. Mark S and complete PQRS. PQRS is the required quadrilateral (Fig 4.9).
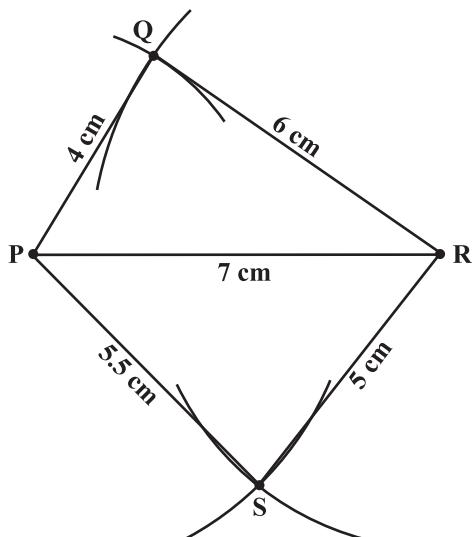
Fig 4.9

THINK, DISCUSS AND WRITE

(i) We saw that 5 measurements of a quadrilateral can determine a quadrilateral uniquely. Do you think any five measurements of the quadrilateral can do this?

(ii) Can you draw a parallelogram BATS where BA = 5 cm, AT = 6 cm and AS = 6.5 cm? Why?

(iii) Can you draw a rhombus ZEAL where ZE = 3.5 cm, diagonal EL = 5 cm? Why?

(iv) A student attempted to draw a quadrilateral PLAY where PL = 3 cm, LA = 4 cm, AY = 4.5 cm, PY = 2 cm and LY = 6 cm, but could not draw it. What is the reason?
[**Hint:** Discuss it using a rough sketch].

EXERCISE 4.1

1. Construct the following quadrilaterals.

   (i) Quadrilateral ABCD.
   AB = 4.5 cm
   BC = 5.5 cm
   CD = 4 cm
   AD = 6 cm
   AC = 7 cm

   (ii) Quadrilateral JUMP
   JU = 3.5 cm
   UM = 4 cm
   MP = 5 cm
   PJ = 4.5 cm
   PU = 6.5 cm

   (iii) Parallelogram MORE
   OR = 6 cm
   RE = 4.5 cm
   EO = 7.5 cm

   (iv) Rhombus BEST
   BE = 4.5 cm
   ET = 6 cm

4.2.2 When two diagonals and three sides are given

When four sides and a diagonal were given, we first drew a triangle with the available data and then tried to locate the fourth point. The same technique is used here.

Example 2: Construct a quadrilateral ABCD, given that BC = 4.5 cm, AD = 5.5 cm, CD = 5 cm the diagonal AC = 5.5 cm and diagonal BD = 7 cm.

Solution:

Here is the rough sketch of the quadrilateral ABCD (Fig 4.10). Studying this sketch, we can easily see that it is possible to draw Δ ACD first (How?).
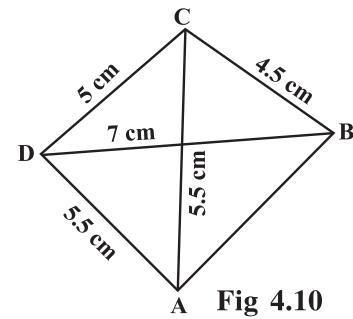
Fig 4.10

Step 1 Draw Δ ACD using SSS construction (Fig 4.11). (We now need to find B at a distance of 4.5 cm from C and 7 cm from D).
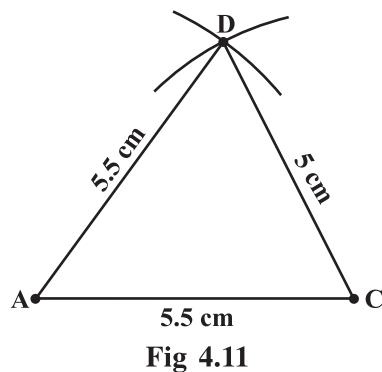
Fig 4.11

Step 2 With D as centre, draw an arc of radius 7 cm. (B is somewhere on this arc) (Fig 4.12).
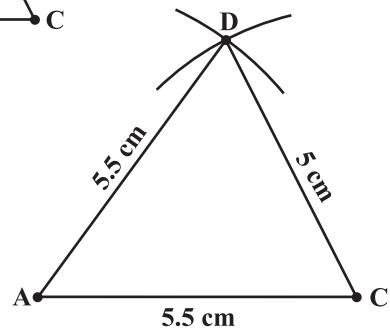
Fig 4.12

Step 3 With C as centre, draw an arc of radius 4.5 cm (B is somewhere on this arc also) (Fig 4.13).
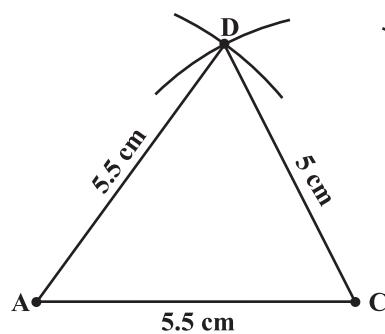
Fig 4.13

Step 4 Since B lies on both the arcs, B is the point intersection of the two arcs. Mark B and complete ABCD. ABCD is the required quadrilateral (Fig 4.14).
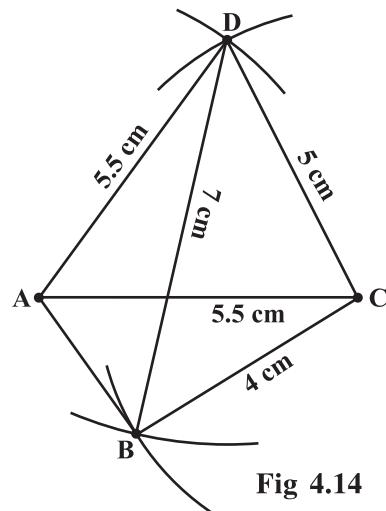

Fig 4.14

THINK, DISCUSS AND WRITE

1. In the above example, can we draw the quadrilateral by drawing △ABD first and then find the fourth point C?

2. Can you construct a quadrilateral PQRS with PQ = 3 cm, RS = 3 cm, PS = 7.5 cm, PR = 8 cm and SQ = 4 cm? Justify your answer.

EXERCISE 4.2

1. Construct the following quadrilaterals.

(i) quadrilateral LIFT

LI = 4 cm

IF = 3 cm

TL = 2.5 cm

LF = 4.5 cm

IT = 4 cm

(ii) Quadrilateral GOLD

OL = 7.5 cm

GL = 6 cm

GD = 6 cm

LD = 5 cm

OD = 10 cm

(iii) Rhombus BEND

BN = 5.6 cm

DE = 6.5 cm

4.2.3 When two adjacent sides and three angles are known

As before, we start with constructing a triangle and then look for the fourth point to complete the quadrilateral.

Example 3: Construct a quadrilateral MIST where MI = 3.5 cm, IS = 6.5 cm, $\angle M = 75°$, $\angle I = 105°$ and $\angle S = 120°$.

Solution:

Here is a rough sketch that would help us in deciding our steps of construction. We give only hints for various steps (Fig 4.15).
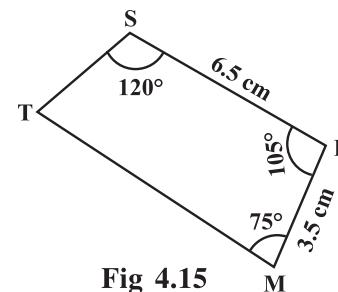
Fig 4.15

Step 1 How do you locate the points? What choice do you make for the base and what is the first step? (Fig 4.16)
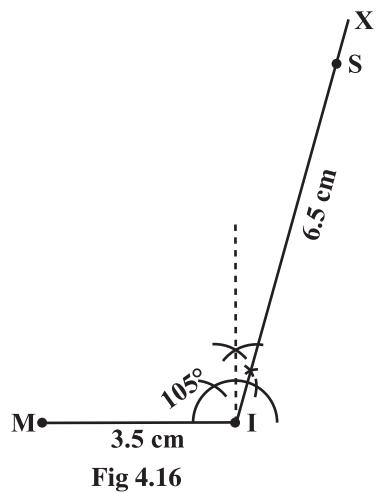
Fig 4.16

Step 2 Make ∠ISY = 120° at S (Fig 4.17).
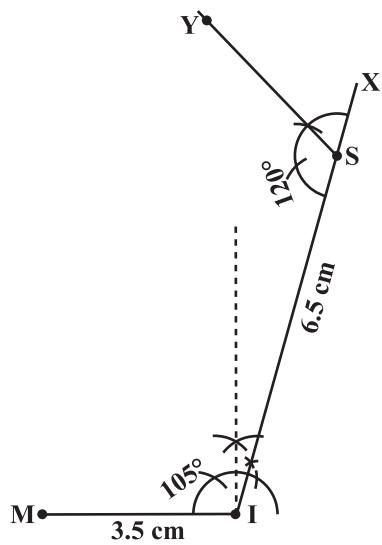
Fig 4.17

Step 3   Make ∠IMZ = 75° at M. (where will SY and MZ meet?) Mark that point as T. We get the required quadrilateral MIST (Fig 4.18).
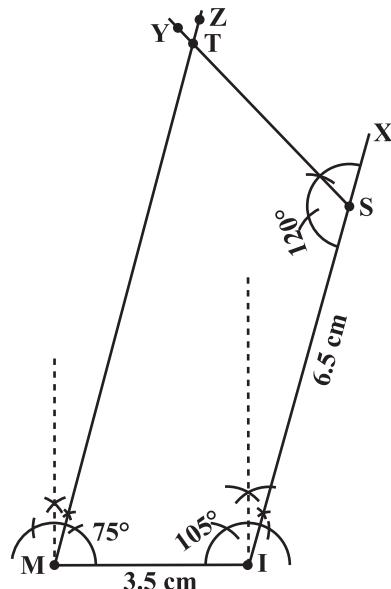

Fig 4.18

THINK, DISCUSS AND WRITE

1. Can you construct the above quadrilateral MIST if we have 100° at M instead of 75°?

2. Can you construct the quadrilateral PLAN if PL = 6 cm, LA = 9.5 cm, ∠P = 75°, ∠L =150° and ∠A = 140°? (**Hint:** Recall angle-sum property).

3. In a parallelogram, the lengths of adjacent sides are known. Do we still need measures of the angles to construct as in the example above?

EXERCISE 4.3

1. Construct the following quadrilaterals.

  (i)   Quadrilateral MORE
        MO = 6 cm
        OR = 4.5 cm
        ∠M = 60°
        ∠O = 105°
        ∠R = 105°

  (ii) Quadrilateral PLAN
        PL = 4 cm
        LA = 6.5 cm
        ∠P = 90°
        ∠A = 110°
        ∠N = 85°

  (iii) Parallelogram HEAR
        HE = 5 cm
        EA = 6 cm
        ∠R = 85°

  (iv) Rectangle OKAY
        OK = 7 cm
        KA = 5 cm

4.2.4 When three sides and two included angles are given

Under this type, when you draw a rough sketch, note carefully the "included" angles in particular.

Example 4: Construct a quadrilateral ABCD, where AB = 4 cm, BC = 5 cm, CD = 6.5 cm and ∠B = 105° and ∠C = 80°.

Solution:

We draw a rough sketch, as usual, to get an idea of how we can start off. Then we can devise a plan to locate the four points (Fig 4.19).
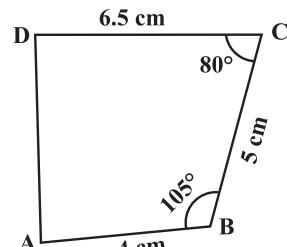
Fig 4.19

Step 1 Start with taking BC = 5 cm on B. Draw an angle of 105° along BX. Locate A 4 cm away on this. We now have B, C and A (Fig 4.20).
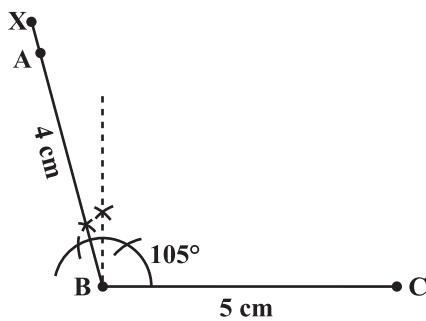
Fig 4.20

Step 2 The fourth point D is on CY which is inclined at 80° to BC. So make ∠BCY = 80° at C on BC (Fig 4.21) .
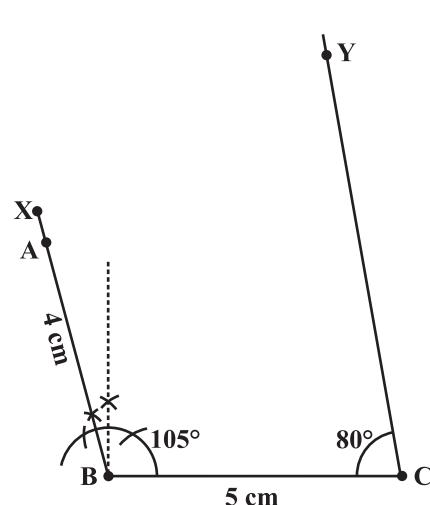
Fig 4.21

Step 3   D is at a distance of 6.5 cm on CY. With
C as centre, draw an arc of length 6.5 cm.
It cuts CY at D (Fig 4.22).
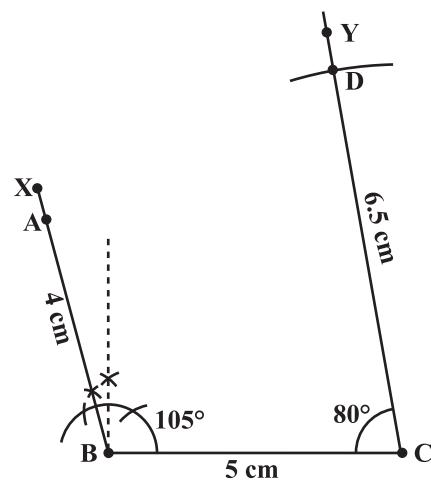

Fig 4.22

Step 4   Complete the quadrilateral ABCD. ABCD is the required quadrilateral (Fig 4.23).
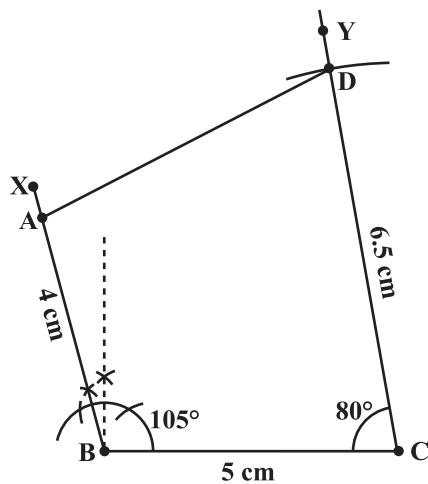

Fig 4.23

THINK, DISCUSS AND WRITE

1. In the above example, we first drew BC. Instead, what could have been be the other starting points?
2. We used some five measurements to draw quadrilaterals so far. Can there be different sets of five measurements (other than seen so far) to draw a quadrilateral? The following problems may help you in answering the question.
   (i) Quadrilateral ABCD with AB = 5 cm, BC = 5.5 cm, CD = 4 cm, AD = 6 cm and ∠B = 80°.
   (ii) Quadrilateral PQRS with PQ = 4.5 cm, ∠P = 70°, ∠Q = 100°, ∠R = 80° and ∠S = 110°.

   Construct a few more examples of your own to find sufficiency/insufficiency of the data for construction of a quadrilateral.

EXERCISE 4.4

1. Construct the following quadrilaterals.

   (i) Quadrilateral DEAR
       DE = 4 cm
       EA = 5 cm
       AR = 4.5 cm
       ∠E = 60°
       ∠A = 90°

   (ii) Quadrilateral TRUE
       TR = 3.5 cm
       RU = 3 cm
       UE = 4 cm
       ∠R = 75°
       ∠U = 120°

4.3 Some Special Cases

To draw a quadrilateral, we used 5 measurements in our work. Is there any quadrilateral which can be drawn with less number of available measurements? The following examples examine such special cases.

Example 5: Draw a square of side 4.5 cm.

Solution: Initially it appears that only one measurement has been given. Actually we have many more details with us, because the figure is a special quadrilateral, namely a square. We now know that each of its angles is a right angle. (See the rough figure) (Fig 4.24)

   This enables us to draw ΔABC using SAS condition. Then D can be easily located. Try yourself now to draw the square with the given measurements.

Rough figure

Fig 4.24

Example 6: Is it possible to construct a rhombus ABCD where AC = 6 cm and BD = 7 cm? Justify your answer.

Solution: Only two (diagonal) measurements of the rhombus are given. However, since it is a rhombus, we can find more help from its properties.

The diagonals of a rhombus are perpendicular bisectors of one another.

   So, first draw AC = 7 cm and then construct its perpendicular bisector. Let them meet at 0. Cut off 3 cm lengths on either side of the drawn bisector. You now get B and D.

   Draw the rhombus now, based on the method described above (Fig 4.25).

Rough figure
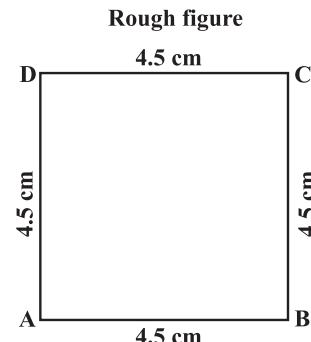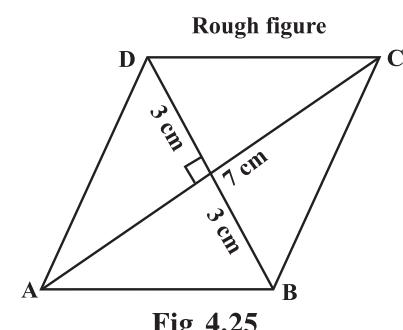
Fig 4.25

TRY THESE

1. How will you construct a rectangle PQRS if you know only the lengths PQ and QR?

2. Construct the kite EASY if AY = 8 cm, EY = 4 cm and SY = 6 cm (Fig 4.26). Which properties of the kite did you use in the process?
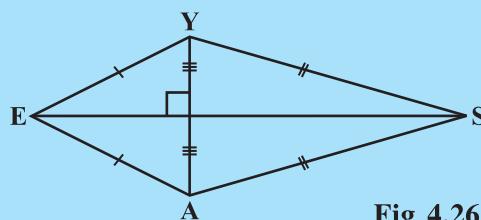

Fig 4.26

EXERCISE 4.5

Draw the following.

1. The square READ with RE = 5.1 cm.
2. A rhombus whose diagonals are 5.2 cm and 6.4 cm long.
3. A rectangle with adjacent sides of lengths 5 cm and 4 cm.
4. A parallelogram OKAY where OK = 5.5 cm and KA = 4.2 cm. Is it unique?

WHAT HAVE WE DISCUSSED?

1. Five measurements can determine a quadrilateral uniquely.
2. A quadrilateral can be constructed uniquely if the lengths of its four sides and a diagonal is given.
3. A quadrilateral can be constructed uniquely if its two diagonals and three sides are known.
4. A quadrilateral can be constructed uniquely if its two adjacent sides and three angles are known.
5. A quadrilateral can be constructed uniquely if its three sides and two included angles are given.

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Data Handling

</div>

5.1 Looking for Information

In your day-to-day life, you might have come across information, such as:

 (a) Runs made by a batsman in the last 10 test matches.

 (b) Number of wickets taken by a bowler in the last 10 ODIs.

 (c) Marks scored by the students of your class in the Mathematics unit test.

 (d) Number of story books read by each of your friends etc.

 The information collected in all such cases is called **data**. Data is usually collected in the context of a situation that we want to study. For example, a teacher may like to know the average height of students in her class. To find this, she will write the heights of all the students in her class, organise the data in a systematic manner and then interpret it accordingly.

 Sometimes, data is represented **graphically** to give a clear idea of what it represents. Do you remember the different types of graphs which we have learnt in earlier classes?

1. **A Pictograph:** Pictorial representation of data using symbols.

🚗 = 100 cars ← One symbol stands for 100 cars		
July	🚗 🚗 🚗(half)	= 250 🚗(half) denotes $\frac{1}{2}$ of 100
August	🚗 🚗 🚗	= 300
September	🚗 🚗 🚗 🚗	= ?

 (i) How many cars were produced in the month of July?

 (ii) In which month were maximum number of cars produced?

2. **A bar graph:** A display of information using bars of uniform width, their heights being proportional to the respective values.

Bar heights give the quantity for each category.

Bars are of equal width with equal gaps in between.
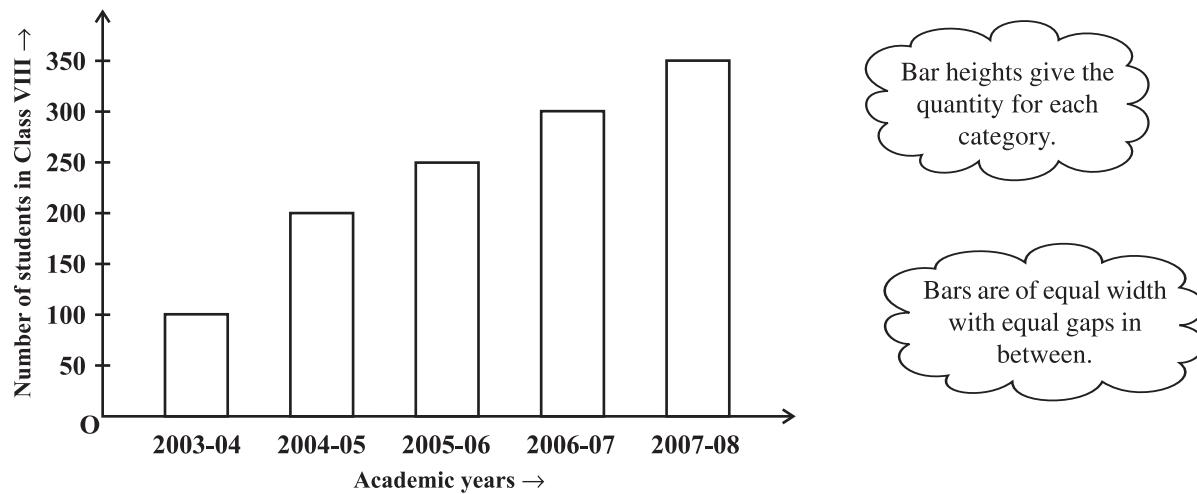
(i)   What is the information given by the bar graph?

(ii) In which year is the increase in the number of students maximum?

(iii) In which year is the number of students maximum?

(iv) State whether true or false:

'The number of students during 2005-06 is twice that of 2003-04.'

3. **Double Bar Graph:** A bar graph showing two sets of data simultaneously. It is useful for the comparison of the data.
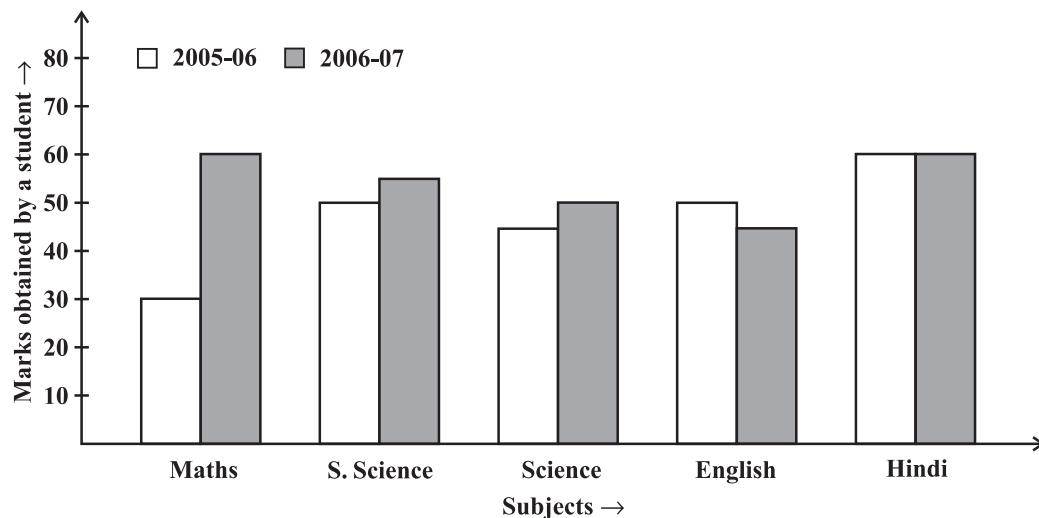
(i)   What is the information given by the double bar graph?

(ii) In which subject has the performance improved the most?

(iii) In which subject has the performance deteriorated?

(iv) In which subject is the performance at par?

THINK, DISCUSS AND WRITE

If we change the position of any of the bars of a bar graph, would it change the information being conveyed? Why?

TRY THESE

Draw an appropriate graph to represent the given information.

1.

Month	July	August	September	October	November	December
Number of watches sold	1000	1500	1500	2000	2500	1500

2.

Children who prefer	School A	School B	School C
Walking	40	55	15
Cycling	45	25	35

3. Percentage wins in ODI by 8 top cricket teams.

Teams	From Champions Trophy to World Cup-06	Last 10 ODI in 07
South Africa	75%	78%
Australia	61%	40%
Sri Lanka	54%	38%
New Zealand	47%	50%
England	46%	50%
Pakistan	45%	44%
West Indies	44%	30%
India	43%	56%

5.2 Organising Data

Usually, data available to us is in an unorganised form called **raw data**. To draw meaningful inferences, we need to organise the data systematically. For example, a group of students was asked for their favourite subject. The results were as listed below:

Art, Mathematics, Science, English, Mathematics, Art, English, Mathematics, English, Art, Science, Art, Science, Science, Mathematics, Art, English, Art, Science, Mathematics, Science, Art.

Which is the most liked subject and the one least liked?

It is not easy to answer the question looking at the choices written haphazardly. We arrange the data in Table 5.1 using tally marks.

Table 5.1

Subject	Tally Marks	Number of Students
Art	ⅢⅡ	7
Mathematics	Ⅲ	5
Science	ⅢⅠ	6
English	ⅠⅠⅠⅠ	4

The number of tallies before each subject gives the number of students who like that particular subject.

This is known as the **frequency** of that subject.

Frequency gives the number of times that a particular entry occurs.

From Table 5.1, Frequency of students who like English is 4

Frequency of students who like Mathematics is 5

The table made is known as **frequency distribution table** as it gives the number of times an entry occurs.

> ## TRY THESE
>
> 1. A group of students were asked to say which animal they would like most to have as a pet. The results are given below:
>   dog, cat, cat, fish, cat, rabbit, dog, cat, rabbit, dog, cat, dog, dog, dog, cat, cow, fish, rabbit, dog, cat, dog, cat, cat, dog, rabbit, cat, fish, dog.
>   Make a frequency distribution table for the same.

5.3 Grouping Data

The data regarding choice of subjects showed the occurrence of each of the entries several times. For example, Art is liked by 7 students, Mathematics is liked by 5 students and so on (Table 5.1). This information can be displayed graphically using a pictograph or a bargraph. Sometimes, however, we have to deal with a large data. For example, consider the following marks (out of 50) obtained in Mathematics by 60 students of Class VIII:

21, 10, 30, 22, 33, 5, 37, 12, 25, 42, 15, 39, 26, 32, 18, 27, 28, 19, 29, 35, 31, 24, 36, 18, 20, 38, 22, 44, 16, 24, 10, 27, 39, 28, 49, 29, 32, 23, 31, 21, 34, 22, 23, 36, 24, 36, 33, 47, 48, 50, 39, 20, 7, 16, 36, 45, 47, 30, 22, 17.

If we make a frequency distribution table for each observation, then the table would be too long, so, for convenience, we make groups of observations say, 0-10, 10-20 and so on, and obtain a frequency distribution of the number of observations falling in each

group. Thus, the frequency distribution table for the above data can be.

Table 5.2

Groups	Tally Marks	Frequency
0-10	||	2
10-20	�l�ⱱ ⱱⱱ	10
20-30	ⱱ ⱱ ⱱ ⱱ |	21
30-40	ⱱ ⱱ ⱱ ||||	19
40-50	ⱱ ||	7
50-60	|	1
	Total	**60**

Data presented in this manner is said to be **grouped** and the distribution obtained is called **grouped frequency distribution**. It helps us to draw meaningful inferences like –

(1) Most of the students have scored between 20 and 40.

(2) Eight students have scored more than 40 marks out of 50 and so on.

Each of the groups 0-10, 10-20, 20-30, etc., is called a **Class Interval** (or briefly a class).

Observe that 10 occurs in both the classes, i.e., 0-10 as well as 10-20. Similarly, 20 occurs in classes 10-20 and 20-30. But it is not possible that an observation (say 10 or 20) can belong simultaneously to two classes. To avoid this, we adopt the convention that the common observation will belong to the higher class, i.e., 10 belongs to the class interval 10-20 (and not to 0-10). Similarly, 20 belongs to 20-30 (and not to 10-20). In the class interval, 10-20, 10 is called the **lower class limit** and 20 is called the **upper class limit**. Similarly, in the class interval 20-30, 20 is the lower class limit and 30 is the upper class limit. Observe that the difference between the upper class limit and lower class limit for each of the class intervals 0-10, 10-20, 20-30 etc., is equal, (10 in this case). This difference between the upper class limit and lower class limit is called the **width** or **size** of the class interval.

TRY THESE

1. Study the following frequency distribution table and answer the questions given below.

Frequency Distribution of Daily Income of 550 workers of a factory

Table 5.3

Class Interval (Daily Income in ₹)	Frequency (Number of workers)
100-125	45
125-150	25

150-175	55
175-200	125
200-225	140
225-250	55
250-275	35
275-300	50
300-325	20
Total	**550**

(i)   What is the size of the class intervals?
(ii)   Which class has the highest frequency?
(iii)   Which class has the lowest frequency?
(iv)   What is the upper limit of the class interval 250-275?
(v)   Which two classes have the same frequency?

2.   Construct a frequency distribution table for the data on weights (in kg) of 20 students of a class using intervals 30-35, 35-40 and so on.

40, 38, 33, 48, 60, 53, 31, 46, 34, 36, 49, 41, 55, 49, 65, 42, 44, 47, 38, 39.

5.3.1 Bars with a difference

Let us again consider the grouped frequency distribution of the marks obtained by 60 students in Mathematics test. (Table 5.4)

Table 5.4

Class Interval	Frequency
0-10	2
10-20	10
20-30	21
30-40	19
40-50	7
50-60	1
Total	**60**

This is displayed graphically as in the adjoining graph (Fig 5.1).

Is this graph in any way different from the bar graphs which you have drawn in Class VII? Observe that, here we have represented the groups of observations (i.e., class intervals)
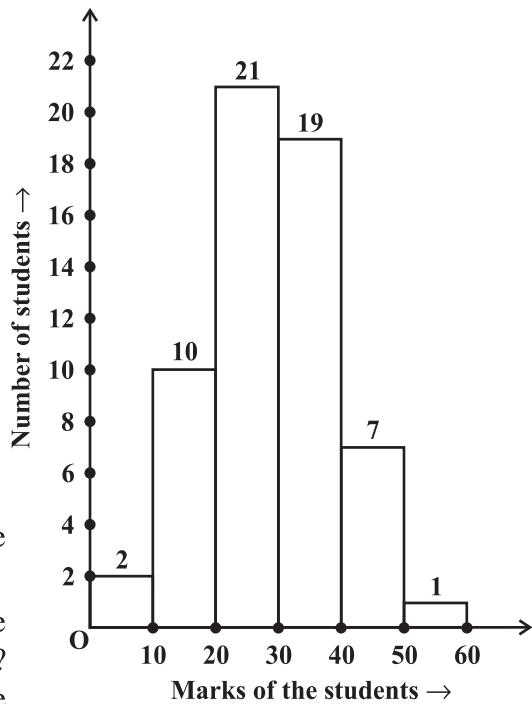

Fig 5.1

on the horizontal axis. The **height** of the bars show the **frequency** of the class-interval. Also, there is no gap between the bars as there is no gap between the class-intervals.

The graphical representation of data in this manner is called a **histogram**. The following graph is another histogram (Fig 5.2).
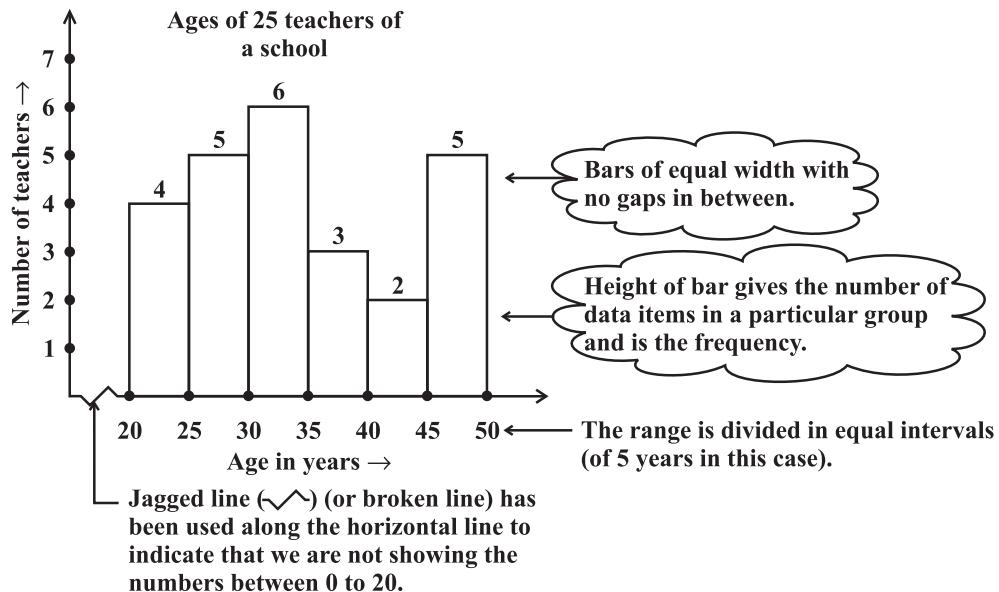
Fig 5.2

From the bars of this histogram, we can answer the following questions:

   (i)   How many teachers are of age 45 years or more but less than 50 years?

   (ii)   How many teachers are of age less than 35 years?

TRY THESE

1.   Observe the histogram (Fig 5.3) and answer the questions given below.
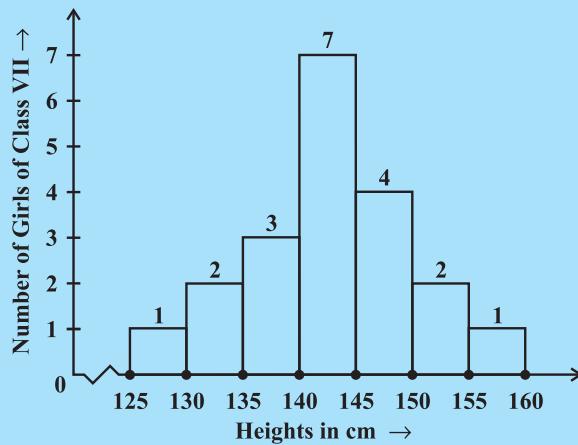

Fig 5.3

   (i)   What information is being given by the histogram?

   (ii)   Which group contains maximum girls?

(iii) How many girls have a height of 145 cms and more?

(iv) If we divide the girls into the following three categories, how many would there be in each?

150 cm and more   — Group A

140 cm to less than 150 cm   — Group B

Less than 140 cm   — Group C

EXERCISE 5.1

1. For which of these would you use a histogram to show the data?

   (a) The number of letters for different areas in a postman's bag.

   (b) The height of competitors in an athletics meet.

   (c) The number of cassettes produced by 5 companies.

   (d) The number of passengers boarding trains from 7:00 a.m. to 7:00 p.m. at a station.

   Give reasons for each.

2. The shoppers who come to a departmental store are marked as: man (M), woman (W), boy (B) or girl (G). The following list gives the shoppers who came during the first hour in the morning:

   W W W G B W W M G G M M W W W W G B M W B G G M W W M M W W

   W M W B W G M W W W W G W M M W W M W G W M G W M M B G G W

   Make a frequency distribution table using tally marks. Draw a bar graph to illustrate it.

3. The weekly wages (in ₹) of 30 workers in a factory are.

   830, 835, 890, 810, 835, 836, 869, 845, 898, 890, 820, 860, 832, 833, 855, 845, 804, 808, 812, 840, 885, 835, 835, 836, 878, 840, 868, 890, 806, 840

   Using tally marks make a frequency table with intervals as 800–810, 810–820 and so on.

4. Draw a histogram for the frequency table made for the data in Question 3, and answer the following questions.

   (i) Which group has the maximum number of workers?

   (ii) How many workers earn ₹ 850 and more?

   (iii) How many workers earn less than ₹ 850?

5. The number of hours for which students of a particular class watched television during holidays is shown through the given graph.

   Answer the following.

   (i) For how many hours did the maximum number of students watch TV?

   (ii) How many students watched TV for less than 4 hours?

(iii)   How many students spent more than 5 hours in watching TV?
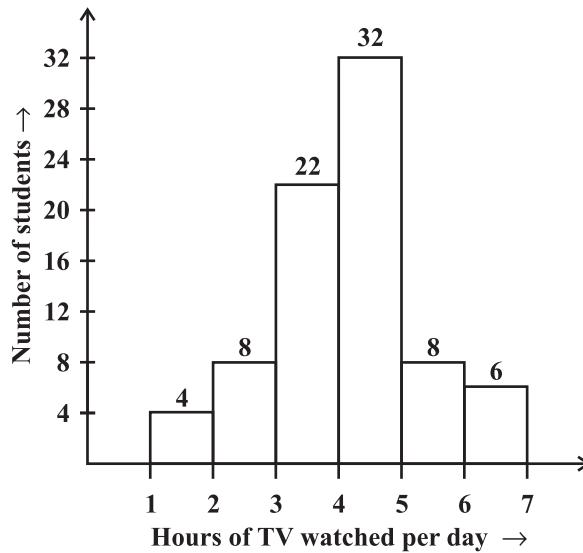

5.4 Circle Graph or Pie Chart

Have you ever come across data represented in circular form as shown (Fig 5.4)?

The time spent by a child during a day   Age groups of people in a town
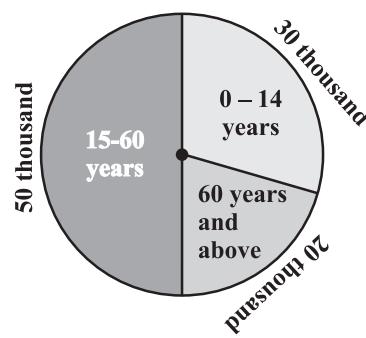
(i)   **Fig 5.4**   (ii)

These are called **circle graphs**. A circle graph shows the relationship between a whole and its parts. Here, the whole circle is divided into sectors. The size of each sector is proportional to the activity or information it represents.

For example, in the above graph, the proportion of the sector for hours spent in sleeping

$$= \frac{\text{number of sleeping hours}}{\text{whole day}} = \frac{8 \text{ hours}}{24 \text{ hours}} = \frac{1}{3}$$

So, this sector is drawn as $\frac{1}{3}$ rd part of the circle. Similarly, the proportion of the sector

for hours spent in school $= \dfrac{\text{number of school hours}}{\text{whole day}} = \dfrac{6 \text{ hours}}{24 \text{ hours}} = \dfrac{1}{4}$

So this sector is drawn $\frac{1}{4}$ th of the circle. Similarly, the size of other sectors can be found.

Add up the fractions for all the activities. Do you get the total as one?

A circle graph is also called a **pie chart**.

TRY THESE

1. Each of the following pie charts (Fig 5.5) gives you a different piece of information about your class. Find the fraction of the circle representing each of these information.

(i)

Girls
50%

Boys
50%

Girls or Boys

(ii)

Walk
40%

20%
Cycle

Bus or car
40%

Transport to school

(iii)

Love

15%
Hate

Love/Hate Mathematics
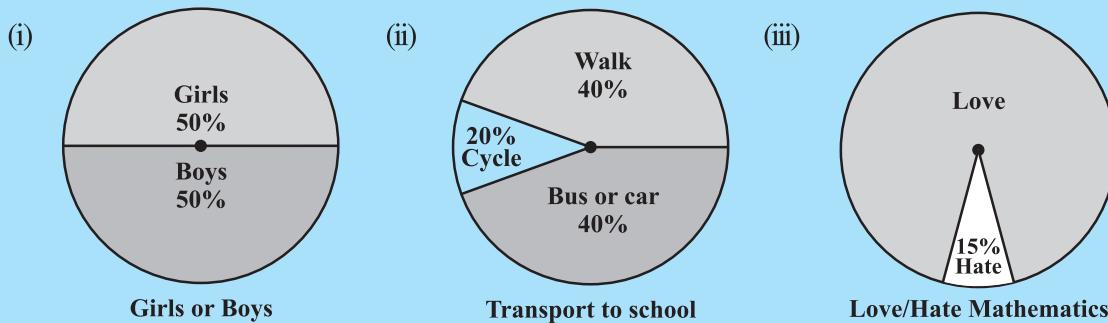
Fig 5.5

2. Answer the following questions based on the pie chart given (Fig 5.6).

    (i) Which type of programmes are viewed the most?

    (ii) Which two types of programmes have number of viewers equal to those watching sports channels?

Informative
10%

News
15%

Sports
25%

Entertainment
50%
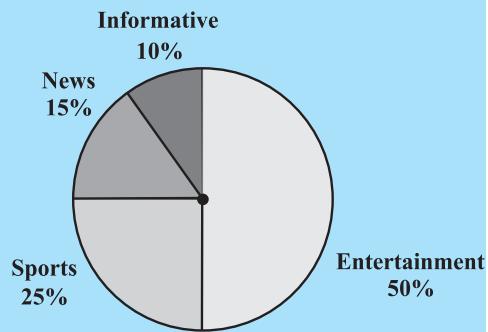
Viewers watching different types of channels on T.V.

Fig 5.6

5.4.1 Drawing pie charts

The favourite flavours of ice-creams for students of a school is given in percentages as follows.

Let us represent this data in a pie chart.

| Flavours	Percentage of students Preferring the flavours
Chocolate	50%
Vanilla	25%
Other flavours	25%

The total angle at the centre of a circle is 360°. The central angle of the sectors will be

a fraction of 360°. We make a table to find the central angle of the sectors (Table 5.5).

Table 5.5

| Flavours	Students in per cent preferring the flavours	In fractions	Fraction of 360°
Chocolate	50%	$\frac{50}{100} = \frac{1}{2}$	$\frac{1}{2}$ of 360° = 180°
Vanilla	25%	$\frac{25}{100} = \frac{1}{4}$	$\frac{1}{4}$ of 360° = 90°
Other flavours	25%	$\frac{25}{100} = \frac{1}{4}$	$\frac{1}{4}$ of 360° = 90°

1. Draw a circle with any convenient radius.
   Mark its centre (O) and a radius (OA).
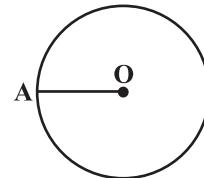
2. The angle of the sector for chocolate is 180°.
   Use the protractor to draw ∠AOB = 180°.
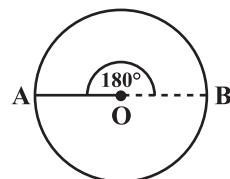
3. Continue marking the remaining sectors.

Example 1: Adjoining pie chart (Fig 5.7) gives the expenditure (in percentage) on various items and savings of a family during a month.

   (i) On which item, the expenditure was maximum?
   (ii) Expenditure on which item is equal to the total savings of the family?
   (iii) If the monthly savings of the family is ₹ 3000, what is the monthly expenditure on clothes?

Solution:

   (i) Expenditure is maximum on food.
   (ii) Expenditure on Education of children is the same (i.e., 15%) as the savings of the family.
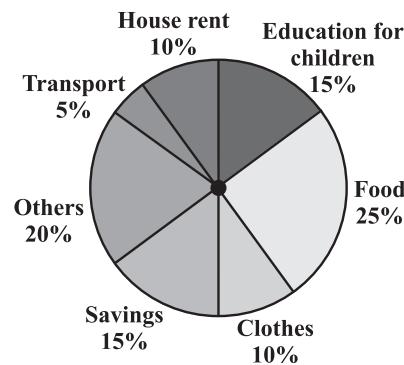
Fig 5.7

(iii) 15% represents ₹ 3000

Therefore, 10% represents ₹ $\dfrac{3000}{15} \times 10 = ₹\, 2000$

Example 2: On a particular day, the sales (in rupees) of different items of a baker's shop are given below.

ordinary bread	: 320
fruit bread	: 80
cakes and pastries	: 160
biscuits	: 120
others	: 40
Total	**: 720**

Draw a pie chart for this data.

Solution: We find the central angle of each sector. Here the total sale = ₹ 720. We thus have this table.

Item	Sales (in ₹)	In Fraction	Central Angle
Ordinary Bread	320	$\dfrac{320}{720} = \dfrac{4}{9}$	$\dfrac{4}{9} \times 360° = 160°$
Biscuits	120	$\dfrac{120}{720} = \dfrac{1}{6}$	$\dfrac{1}{6} \times 360° = 60°$
Cakes and pastries	160	$\dfrac{160}{720} = \dfrac{2}{9}$	$\dfrac{2}{9} \times 360° = 80°$
Fruit Bread	80	$\dfrac{80}{720} = \dfrac{1}{9}$	$\dfrac{1}{9} \times 360° = 40°$
Others	40	$\dfrac{40}{720} = \dfrac{1}{18}$	$\dfrac{1}{18} \times 360° = 20°$

Now, we make the pie chart (Fig 5.8):
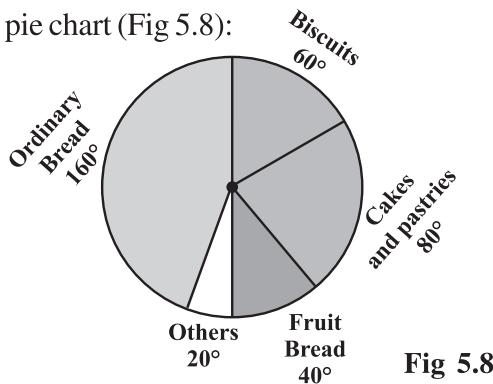
Fig 5.8

TRY THESE

Draw a pie chart of the data given below.

The time spent by a child during a day.

Sleep — 8 hours

School — 6 hours

Home work — 4 hours

Play — 4 hours

Others — 2 hours

THINK, DISCUSS AND WRITE

Which form of graph would be appropriate to display the following data.

1. **Production of food grains of a state.**

Year	2001	2002	2003	2004	2005	2006
Production (in lakh tons)	60	50	70	55	80	85

2. **Choice of food for a group of people.**

Favourite food	Number of people
North Indian	30
South Indian	40
Chinese	25
Others	25
Total	**120**

3. **The daily income of a group of a factory workers.**

| Daily Income (in Rupees)	Number of workers (in a factory)
75-100	45
100-125	35
125-150	55
150-175	30
175-200	50
200-225	125
225-250	140
Total	**480**

EXERCISE 5.2

1. A survey was made to find the type of music that a certain group of young people liked in a city. Adjoining pie chart shows the findings of this survey.

   From this pie chart answer the following:
   (i) If 20 people liked classical music, how many young people were surveyed?
   (ii) Which type of music is liked by the maximum number of people?
   (iii) If a cassette company were to make 1000 CD's, how many of each type would they make?

2. A group of 360 people were asked to vote for their favourite season from the three seasons rainy, winter and summer.
   (i) Which season got the most votes?
   (ii) Find the central angle of each sector.
   (iii) Draw a pie chart to show this information.

Season		No. of votes
Summer		90
Rainy		120
Winter		150

3. Draw a pie chart showing the following information. The table shows the colours preferred by a group of people.

Colours	Number of people
Blue	18
Green	9
Red	6
Yellow	3
Total	**36**

Find the proportion of each sector. For example, Blue is $\dfrac{18}{36} = \dfrac{1}{2}$; Green is $\dfrac{9}{36} = \dfrac{1}{4}$ and so on. Use this to find the corresponding angles.

4. The adjoining pie chart gives the marks scored in an examination by a student in Hindi, English, Mathematics, Social Science and Science. If the total marks obtained by the students were 540, answer the following questions.
   (i) In which subject did the student score 105 marks?
   (**Hint:** for 540 marks, the central angle = 360°. So, for 105 marks, what is the central angle?)
   (ii) How many more marks were obtained by the student in Mathematics than in Hindi?
   (iii) Examine whether the sum of the marks obtained in Social Science and Mathematics is more than that in Science and Hindi.
   (**Hint:** Just study the central angles).
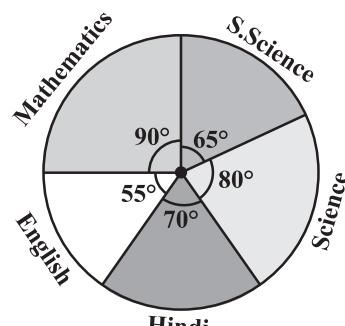
5. The number of students in a hostel, speaking different languages is given below. Display the data in a pie chart.

Language	Hindi	English	Marathi	Tamil	Bengali	Total
Number of students	40	12	9	7	4	72

5.5 Chance and Probability

Sometimes it happens that during rainy season, you carry a raincoat every day and it does not rain for many days. However, by chance, one day you forget to take the raincoat and it rains heavily on that day.

Sometimes it so happens that a student prepares 4 chapters out of 5, very well for a test. But a major question is asked from the chapter that she left unprepared.

Everyone knows that a particular train runs in time but the day you reach well in time it is late!

You face a lot of situations such as these where you take a chance and it does not go the way you want it to. Can you give some more examples? These are examples where the chances of a certain thing happening or not happening are not equal. The chances of the train being in time or being late are not the same. When you buy a ticket which is wait listed, you do take a chance. You hope that it might get confirmed by the time you travel.

We however, consider here certain experiments whose results have an equal chance of occurring.

5.5.1 Getting a result

You might have seen that before a cricket match starts, captains of the two teams go out to toss a coin to decide which team will bat first.

What are the possible results you get when a coin is tossed? Of course, Head or Tail.

Imagine that you are the captain of one team and your friend is the captain of the other team. You toss a coin and ask your friend to make the call. Can you control the result of the toss? Can you get a head if you want one? Or a tail if you want that? No, that is not possible. Such an experiment is called a **random experiment**. Head or Tail are the two **outcomes** of this experiment.

> ### TRY THESE
>
> 1. If you try to start a scooter, what are the possible outcomes?
> 2. When a die is thrown, what are the six possible outcomes?

3. When you spin the wheel shown, what are the possible outcomes? (Fig 5.9) List them.

   (Outcome here means the sector at which the pointer stops).
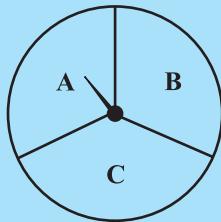
Fig 5.9
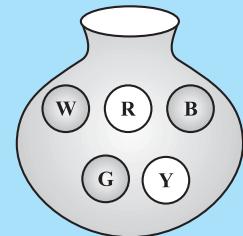
Fig 5.10

4. You have a bag with five identical balls of different colours and you are to pull out (draw) a ball without looking at it; list the outcomes you would get (Fig 5.10).

THINK, DISCUSS AND WRITE

In throwing a die:

- Does the first player have a greater chance of getting a six?
- Would the player who played after him have a lesser chance of getting a six?
- Suppose the second player got a six. Does it mean that the third player would not have a chance of getting a six?

5.5.2 Equally likely outcomes:

A coin is tossed several times and the number of times we get head or tail is noted. Let us look at the result sheet where we keep on increasing the tosses:

Number of tosses	Tally marks (H)	Number of heads	Tally mark (T)	Number of tails
50	NNN NNN NNN NNN NNN II	27	NNN NNN NNN NNN III	23
60	NNN NNN NNN NNN NNN III	28	NNN NNN NNN NNN NNN NNN II	32
70	...	33	...	37
80	...	38	...	42
90	...	44	...	46
100	...	48	...	52

Observe that as you increase the number of tosses more and more, the number of heads and the number of tails come closer and closer to each other.

This could also be done with a die, when tossed a large number of times. Number of each of the six outcomes become almost equal to each other.

In such cases, we may say that the different outcomes of the experiment are equally likely. This means that each of the outcomes has the same chance of occurring.
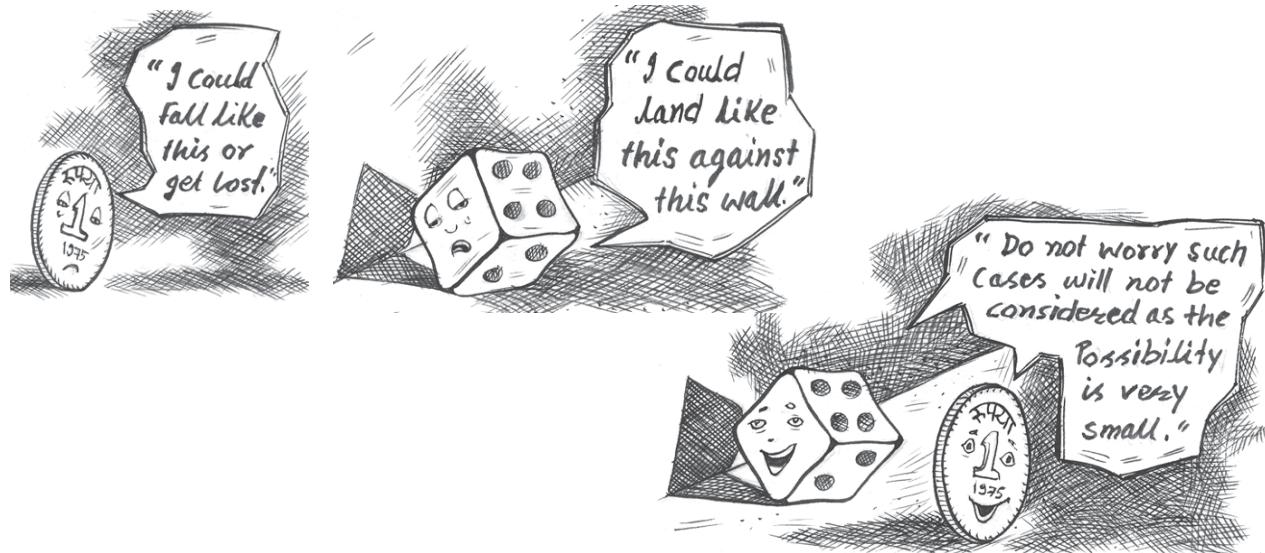
5.5.3 Linking chances to probability

Consider the experiment of tossing a coin once. What are the outcomes? There are only two outcomes – Head or Tail. Both the outcomes are equally likely. Likelihood of getting a head is one out of two outcomes, i.e., $\frac{1}{2}$. In other words, we say that the probability of getting a head $= \frac{1}{2}$. What is the probability of getting a tail?

Now take the example of throwing a die marked with 1, 2, 3, 4, 5, 6 on its faces (one number on one face). If you throw it once, what are the outcomes?

The outcomes are: 1, 2, 3, 4, 5, 6. Thus, there are six equally likely outcomes.

What is the probability of getting the outcome '2'?

It is $\dfrac{1}{6}$ $\begin{array}{l}\leftarrow \text{ Number of outcomes giving 2}\\ \leftarrow \text{ Number of equally likely outcomes.}\end{array}$

What is the probability of getting the number 5? What is the probability of getting the number 7? What is the probability of getting a number 1 through 6?

5.5.4 Outcomes as events

Each outcome of an experiment or a collection of outcomes make an **event**.

For example in the experiment of tossing a coin, getting a Head is an event and getting a Tail is also an event.

In case of throwing a die, getting each of the outcomes 1, 2, 3, 4, 5 or 6 is an event.

Is getting an even number an event? Since an even number could be 2, 4 or 6, getting an even number is also an event. What will be the probability of getting an even number?

It is $\dfrac{3}{6}$ ← Number of outcomes that make the event
← Total number of outcomes of the experiment.

Example 3: A bag has 4 red balls and 2 yellow balls. (The balls are identical in all respects other than colour). A ball is drawn from the bag without looking into the bag. What is probability of getting a red ball? Is it more or less than getting a yellow ball?

Solution: There are in all (4 + 2 =) 6 outcomes of the event. Getting a red ball consists of 4 outcomes. (Why?)

Therefore, the probability of getting a red ball is $\dfrac{4}{6} = \dfrac{2}{3}$. In the same way the probability of getting a yellow ball $= \dfrac{2}{6} = \dfrac{1}{3}$ (Why?). Therefore, the probability of getting a red ball is more than that of getting a yellow ball.

TRY THESE

Suppose you spin the wheel

1. (i) List the number of outcomes of getting a green sector and not getting a green sector on this wheel (Fig 5.11).

   (ii) Find the probability of getting a green sector.

   (iii) Find the probability of not getting a green sector.
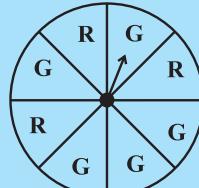
Fig 5.11

### 5.5.5 Chance and probability related to real life

We talked about the chance that it rains just on the day when we do not carry a rain coat.

What could you say about the chance in terms of probability? Could it be one in 10 days during a rainy season? The probability that it rains is then $\dfrac{1}{10}$. The probability that it does not rain $= \dfrac{9}{10}$. (Assuming raining or not raining on a day are equally likely)

The use of probability is made in various cases in real life.

1. To find characteristics of a large group by using a small part of the group.

   For example, during elections 'an exit poll' is taken. This involves asking the people whom they have voted for, when they come out after voting at the centres which are chosen off hand and distributed over the whole area. This gives an idea of chance of winning of each candidate and predictions are made based on it accordingly.

2. Metrological Department predicts weather by observing trends from the data over many years in the past.

EXERCISE 5.3

1. List the outcomes you can see in these experiments.

   (a) Spinning a wheel
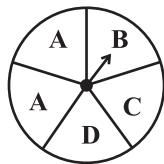
   (b) Tossing two coins together

2. When a die is thrown, list the outcomes of an event of getting

   (i) (a) a prime number (b) not a prime number.

   (ii) (a) a number greater than 5 (b) a number not greater than 5.

3. Find the.

   (a) Probability of the pointer stopping on D in (Question 1-(a))?

   (b) Probability of getting an ace from a well shuffled deck of 52 playing cards?

   (c) Probability of getting a red apple. (See figure below)
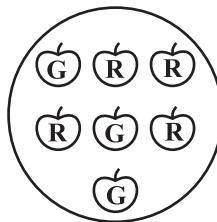
4. Numbers 1 to 10 are written on ten separate slips (one number on one slip), kept in a box and mixed well. One slip is chosen from the box without looking into it. What is the probability of .

   (i) getting a number 6?

   (ii) getting a number less than 6?

   (iii) getting a number greater than 6?

   (iv) getting a 1-digit number?

5. If you have a spinning wheel with 3 green sectors, 1 blue sector and 1 red sector, what is the probability of getting a green sector? What is the probability of getting a non blue sector?

6. Find the probabilities of the events given in Question 2.

WHAT HAVE WE DISCUSSED?

1. Data mostly available to us in an unorganised form is called **raw data**

2. In order to draw meaningful inferences from any data, we need to organise the data systematically.

3. **Frequency** gives the number of times that a particular entry occurs.

4. Raw data can be 'grouped' and presented systematically through 'grouped frequency distribution'.

5. Grouped data can be presented using **histogram**. Histogram is a type of bar diagram, where the class intervals are shown on the horizontal axis and the heights of the bars show the frequency of the class interval. Also, there is no gap between the bars as there is no gap between the class intervals.

6. Data can also presented using **circle graph** or **pie chart**. A circle graph shows the relationship between a whole and its part.

7. There are certain experiments whose outcomes have an equal chance of occurring.

8. A **random experiment** is one whose outcome cannot be predicted exactly in advance.

9. Outcomes of an experiment are **equally likely** if each has the same chance of occurring.

10. **Probability of an event** $= \dfrac{\text{Number of outcomes that make an event}}{\text{Total number of outcomes of the experiment}}$, when the outcomes are equally likely.

11. One or more outcomes of an experiment make an **event**.

12. Chances and probability are related to real life.